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ORIGINAL - SURVEY OR EXPOSITION

A review of operations research models in invasive species management: state of the art, challenges, and future directions

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Abstract Invasive species are a major threat to the economy, the environment, health, and thus human well-being. The international community, including the United Nations' Global Invasive Species Program (GISP), National Invasive Species Council (NISC), and Center for Invasive Species Management (CISM), has called for a rapid control of invaders in order to minimize their adverse impacts. The effective management of invasive species is a highly complex problem requiring the development of decision tools that help managers prioritize actions most efficiently by considering corresponding bio-economic costs, impacts on ecosystems, and benefits of control. Operations research methods, such as mathematical programming models, are powerful tools for evaluating different management strategies and providing optimal decisions for allocating limited resources to control invaders. In this paper, we summarize the mathematical models applied to optimize invasive species prevention, surveillance, and control. We first define key concepts in invasive species management (ISM) in a framework that characterizes biological invasions, associated economic and environmental costs, and their management. We then present a spatio-temporal optimization model that illustrates various biological and economic aspects of an ISM problem. Next, we classify the relevant literature with respect to modeling methods: optimal control, stochastic dynamic programming, linear programming, mixed-integer programming, simulation models, and others. We further classify the ISM models with respect to the solution method used, their focus and objectives, and the specific application considered. We discuss limitations of the existing research and provide several directions for further research in optimizing ISM planning. Our review highlights the fact that operations research could play a key role in ISM and environmental decision-making, in particular closing the gap between the decision-support needs of managers and the decision-making tools currently available to management.

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1 Introduction

Invasive species have become a global hazard to the environment, society, and the economy—threatening biodiversity and increasing environmental degradation (Wilcove et al. 1998; Levine et al. 2003); increasing health problems and jeopardizing life, property, and safety due to increased fire risks (Bowers et al. 2006; McDonald and McPherson 2013); and reducing the value of agricultural products and costing U.S. taxpayers billions of dollars annually (Pimentel et al. 2005). The National Invasive Species Council (NISC) stresses the importance of rapid response to controlling new invasions before they spread (NISC 2001).

An invasive species is a species defined as follows: "(1) that is non-native to the ecosystem under consideration and (2) whose introduction causes or is likely to cause economic or environmental harm or harm to human health" (Executive Order 13112; U.S. Department of Interiror 1999). Examples of invasive species that have severe impacts on native ecosystems costing billions of dollars each year include the emerald ash borer (*Agrilus planipennis* Fairmaire) (EAB), zebra mussel (*Dreissena*), and Burmese python (*Python bivittatus*). Invaders also spread disease that can be devastating to human health (e.g., the Asian tiger mosquito can lead to more than 20 diseases, including yellow fever and malaria) (Juliano and Philip Lounibos 2005), and some threaten human safety (e.g., deer-vehicle collisions) (Austin et al. 2009; Dolman and Wäber 2008) and food security (Pejchar and Mooney 2009), and reduce land and water recreational opportunities (Southern Arizona Buffelgrass Coordination Center 2011). Consequently, their effective control is critical for the well-being of people (Pejchar and Mooney 2009).

Controlling invasive species over a spatial scale and extended periods of time is a fundamental and challenging problem. Each invasive species shows distinct population dynamics, such as varying growth and dispersal rates, and has different impacts on the ecosystem. A spatially explicit representation of the problem is essential because functions of ecosystems and the growth and dispersal of the invasive species are spatial in nature. Furthermore, invasive species spread over time in unpredictable ways due to the variability of human vectors of transport and environmental factors.

Management of invasive species involves prevention, surveillance, and control strategies. While control strategies focus on the removal of an invasive population, prevention strategies, such as quarantine and inspection, can avoid the introduction of new invasions at the onset, and surveillance increases the chance of early detection, thus improving the chance of successful eradication (Rejmánek and Pitcairn 2002; Rout et al. 2011). However, resources for managing invaders (e.g., funding, labor, rapid-response teams, volunteer groups) are usually limited. Given different biological properties, the invasive species management (ISM) problem can be defined as the problem of determining how to allocate limited resources among different management efforts (prevention, surveillance, and control) over space and time, with the management objective of minimizing economic and environmental damage from invasive species as well as the cost of management. ISM is a resource allocation problem, which is shown to be NP-complete in the strong sense (Kellerer et al. 2004).

The intricacy of the ISM problem requires the development of bio-economic models that help managers decide on the most efficient management strategies by jointly considering the biological features of population dynamics and the economic costs and benefits (Born et al.

2005; Buhle et al. 2005; Dana et al. 2014; Perrings et al. 2000; Pimentel 2011). Operations research (OR) methods, such as mathematical programming (MP) models, are powerful tools for evaluating different intervention strategies and providing optimal decisions for allocating limited resources to control invaders. Thus, OR could play a key role in the field of ISM planning, in addition to its important role in the fields of transportation, energy, telecommunications, and manufacturing. In particular, the improved solvability of optimization models with efficient algorithms and software makes OR a strong alternative to simpler ecological models and simulation methods.

The relevant papers in the literature are spread across a broad range of disciplines, such as ecology, biological conservation, environmental management, forestry, and resource economics. While OR models have been widely used as a tool to study the ISM problem in various disciplines, there is no evaluative review of those models from an OR perspective, identifying the current state-of-the art and presenting related research needs and directions. Our review paper closes this research gap by synthesizing the disparate ISM literature published to date, providing a critical review of the state of the art in modeling and optimization of ISM resource allocation problems, and identifying future research directions.

The remainder of this paper is organized as follows: Sect. 2 discusses the review methodology and contributions. In Sect. 3, a framework to characterize biological invasions and their management is outlined, and an overview of the invasion impacts on ecosystems and their services is presented. In Sect. 4, we present an optimization model for controlling invasive species to highlight the biological and economic components of ISM as a spatio-temporal resource allocation problem. Section 5 deals with the classification and analysis of selected literature regarding ISM with respect to biological, economic, model, and mathematical complexity, and analyzes studies with respect to their focus, application areas, and main results. In Sect. 5, we also discuss various OR modeling and solution approaches in the ISM literature. Finally, Sect. 6 presents some concluding remarks and suggestions for future research directions.

2 Review methodology and contribution

This section presents the procedures that we followed in our systematic literature review and paper contributions. In this review, we focus on the mathematical modeling-based literature that addresses the "invasive species management" problem. Thus, we limit our selection to articles that describe applications of mathematical optimization models and methodologies to invasive species management and resource allocation.

2.1 Review methodology

In this paper, we perform a systematic literature review similar to the review methods outlined by Tranfield et al. (2003), Denyer and Tranfield (2009), and Pullin and Stewart (2006). A systematic literature review overcomes the perceived weaknesses of a narrative review by providing a scientific and transparent process of the reviewers' decisions, procedures, and conclusions with the aim of reducing bias and improving repeatability (Tranfield et al. 2003; Pullin and Stewart 2006). We adopt the evidence-informed literature review methodology, which is based on a five-step approach: question formulation, electronic literature search, study selection and evaluation, classification of studies and synthesis, and reporting results and outcomes of review (Denyer and Tranfield 2009; Wong et al. 2015). Figure 1 provides a summary of the procedures for our systematic literature review.

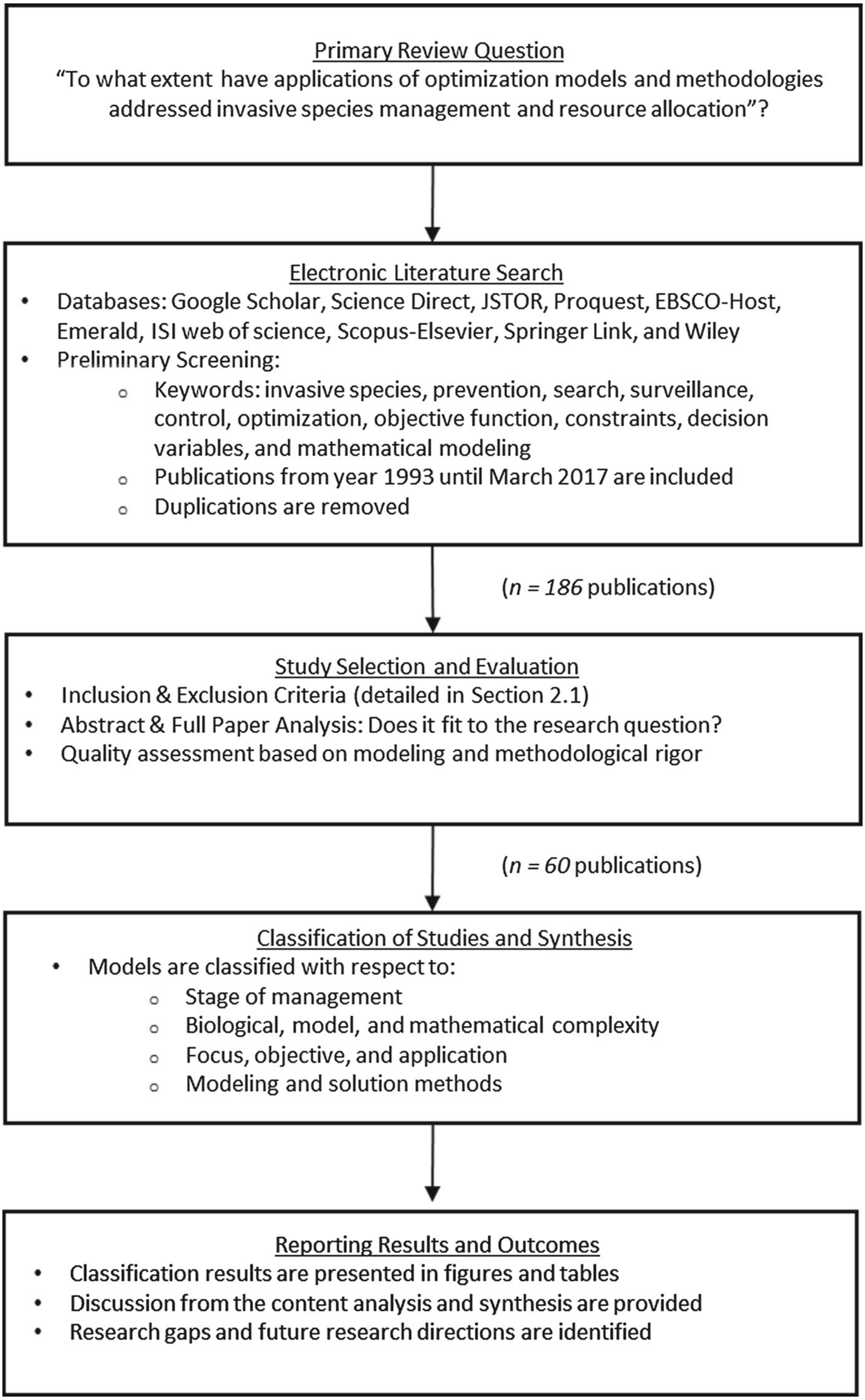


Fig. 1 Steps of systematic literature review (adapted from Denyer and Tranfield 2009 and Wong et al. 2015)

Primary review question The primary review question is: "To what extent have applications of optimization models and methodologies addressed invasive species management and resource allocation?" Answers to this question enable the identification of the scope and contents of the current state-of-the art, the key limitations of existing research as well as future research directions in ISM.

Electronic literature search In the next step we located the relevant literature by identifying a search database and search keywords. Because ISM is multidisciplinary in nature, the related papers are scattered across various journals. Electronic journal databases, including Google Scholar, Science Direct, JSTOR, Proquest, EBSCO-Host, Emerald, ISI web of science, Scopus-Elsevier, Springer Link, and Wiley, were searched for applications of optimization models and methodologies to invasive species management using keywords including "invasive species management," "invasive species," "prevention," "search," "surveillance," "control," "optimization," "objective function," "constraints," "decision variables," and "mathematical modeling."

Study selection and evaluation The search process resulted in more than 8000 references, including refereed journal articles, conference proceedings, dissertations, unpublished works, and books, which we systematically screened for inclusion in the review (Pullin and Stewart 2006; Ho et al. 2015). Based on the database and keyword search, most papers were excluded based on their titles because they were irrelevant to our study. Thus, by filtering the papers using their titles, we identified 186 potentially relevant articles.

Inclusion criteria: We reviewed the abstract and full text of those 186 articles to determine studies to be included in the analysis, classification, and synthesis using five inclusion criteria (a) papers should be written in the English language; (b) they must include decision variables modeling population dynamics as we focus our attention to mathematical models that explicitly incorporate some aspects of biological realism; (c) they must include decision variables modeling control and/or surveillance and/or prevention; (d) they clearly define constraints and an objective function in a mathematical model; and (e) they focus on invasive terrestrial or aquatic plants or animals, or a hypothetical invader.

Exclusion criteria: Articles were excluded if they did not meet one or more of the inclusion criteria. We restricted our search to papers published from year 1993 until March 2017. To further limit the number of publications, we excluded any paper without a mathematical model. Papers with a highly ecological rather than mathematical optimization perspective (e.g., field experiments, empirical studies, statistical, and simulation modeling papers, etc.) were also excluded because our review question concentrates on mathematical optimization studies in invasive species management. We assessed the full text of all publications with relevant titles and abstracts. We excluded duplicate records, i.e., multiple papers that describe the same type of model. We also excluded papers describing mathematical optimization of epidemic pathogens because the review of the literature on epidemic diseases would be another study in itself.

Two co-authors and two peer reviewers contributed to the paper selection process so that no important article from the total of 186 articles was missed. Because these researchers have experience researching and publishing papers in the area, they suggested additional papers that were not formerly included in the literature search.

The abstracts and full texts of a total of 186 articles were carefully reviewed for relevance and contribution to the selected domains covered by this review. After reading those articles, eliminating non-optimization papers, and performing content analysis for answering the review question, the total number of articles was reduced to 60. Thus, of all screened papers, we selected and summarized 60 articles for classification, detailed analysis, and synthesis. The quality assessment of studies was performed based on the rigor of the mathematical model and optimization method applied in the study. While classification was restricted to 60 articles, a total of 180 related papers were cited in this review to describe the components of an ISM framework and to contribute to the synthesis and discussion.

Categorization and reporting results The 60 selected papers were categorized with respect to their contributions in theory and application, similar to the study of Dubey et al. (2017). From a theoretical perspective, we classified ISM papers based on their biological and mathematical complexity as well as modeling and solution methods [e.g., optimal control, stochastic dynamic programming (SDP), linear programming (LP)]. From an application perspective, papers were classified based on the stage of management (e.g., prevention, search, control), specific application they consider (e.g., plant, pest, aquatic), and species name and location (e.g., buffelgrass, Arizona). We also classified papers based on their focus, objectives, and specific conclusions. Paper categorizations were presented in tables and figures to aid understanding. We also provided a detailed discussion from content analysis and synthesis of the selected papers. Finally, we discussed limitations of the existing research and provided several directions for further research in optimizing ISM planning and resource allocation.

2.2 Paper contributions

Several earlier literature reviews have been conducted on various aspects of modeling invasive species management. Olson (2006) reviews the literature on the economics of invasive species management with applications to terrestrial invasive species, in particular. Hastings et al. (2005) review and synthesize studies on the spread of invasive species, by focusing on empirical and statistical approaches as well as data collection. Epanchin-Niell and Hastings (2010) review studies that address the economics of optimal control of established invasive species from the perspective of invasion dynamics, cost of control efforts, and a monetary measure of invasion damages. Dana et al. (2014) review decision tools available for managing biological invasions, and classify studies with respect to geographical focus, habitats, and taxonomic groups. Billionnet (2013) reviews mathematical optimization studies in biodiversity conservation and briefly discusses developments in optimization for fighting against invasive species.

To our knowledge, no review comprehensively analyzes and classifies mathematical methods in ISM from an OR perspective. Our review also differs in purpose, because we seek to assess the models with respect to their computational complexity and the solution methodologies to tackle the ISM problem. To conclude, a comprehensive survey of mathematical models on invasive species management is necessary and useful to guide practitioners and researchers engaged in biological conservation, environmental management, forestry, resource economics, as well as OR. Further, policy makers involved in ISM will benefit from this comprehensive review because OR models of ISM are the foundation of decision support.

The contributions of this paper are summarized as follows:

- To provide a review of the relevant literature while considering several important dimensions not systematically reviewed before, such as the bio-economic modeling characteristics, problem complexity, type of application, spatio-temporal size of the instances studied, and methods used to solve these models.

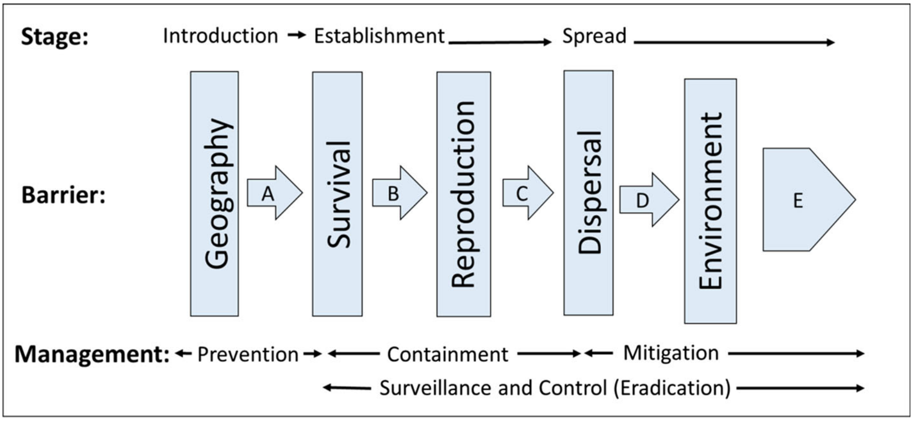
- To define key concepts in ISM in a framework that characterizes biological invasions, associated economic and environmental costs, and their management; and present a comprehensive optimization model to highlight all these dimensions of ISM.
- To summarize the selected articles with respect to their focus, management objective, and main results.
- To investigate to what extent applications of optimization models and methodologies have addressed invasive species management and resource allocation.
- To describe the key imitations of existing research and challenges of mathematical modeling, and to discuss the gap between theoretical research and practical application.
- To propose opportunities for the OR community to make significant future contributions toward solving ISM problems.

3 A framework to characterize biological invasions and their management

Research to improve our understanding of human-mediated plant and animal invasions has grown exponentially over the last half century (Gurevitch et al. 2011), and biologists concerned with different taxa have adopted different frameworks to describe the invasion process. Most plant ecologists adopt a framework that views invasions as a series of barriers that species must overcome in order to establish and spread (Richardson et al. 2000). Most animal ecologists adopt a framework that views invasions as a series of stages that species must pass through on the pathway from native to invasive alien (Williamson 1996; Holmes et al. 2014). Blackburn et al. (2011) merge the plant and animal frameworks into a single unified framework designed to apply to all human-mediated invasions, and we use this framework to synthesize applications of OR in invasive species management.

The unified framework, which we simplify for our purpose, recognizes that the invasion process can be divided into a series of stages, and in each stage, barriers need to be overcome for populations to move to the next stage (Fig. 2).

The process begins with the *Introduction* (sometimes called arrival) stage in which individuals of a species are transported across a geographical barrier beyond the limits of their native range and arrive at a new place. For example, in the horticultural trade, live plants are grown in Central America and imported to the United States, and those imports may be vectors for non-native insects and diseases to the U.S. During the *Introduction* stage, non-native individuals may be intentionally placed in captivity or cultivation or they may be accidentally introduced into a new environment (arrow A). During the *Establishment* stage, introduced individuals must overcome barriers of survival (arrow B) and reproduction (arrow C). Failure to survive or reproduce can result from factors associated with the species (e.g., reproductive rate or specialism), the location (e.g., presence of enemies or mutualists), stochastic features of the individual introduction event (especially propagule pressure), or their interaction (e.g., species location, such as climate matching). While individuals in an introduced population might be able to survive and reproduce in the exotic environment, the population can still fail to establish because the long-term population growth rate is negative. During the *Spread* stage, the population must overcome barriers to dispersal (Quick et al. 2017). If individuals can disperse to locations away from the point of introduction (arrow D), then they must overcome new environmental barriers to survive and reproduce. Thus, a spreading population faces multiple establishment events under an increasing range of environmental conditions. If successful (arrow E), then the population becomes fully invasive, with individuals dispersing, surviving, and reproducing at multiple sites across a range of habitats. The framework



A. Individuals transported beyond limits of native range and directly introduced into novel environment
B. Individuals surviving in the wild in locations where introduced, no reproduction
C. Self-sustaining population in the wild, with individuals surviving and reproducing in locations where introduced
D. Self-sustaining population in the wild, with individuals dispersing, surviving and reproducing a significant distance from points of introductior
E. Fully invasive species, with individuals dispersing, surviving and reproducing at multiple sites across a range of habitats

Fig. 2 Framework to characterize biological invasions and their management (adapted from Blackburn et al. 2011)

recognizes that a non-native population can fail to become an invader because it fails to pass any one of the barriers at any stage of the invasion process. Finally, it is important to note that several populations of an alien species can be present at different locations at different points on the framework. Therefore, reference to position on the invasion framework with respect to any given species should be temporally and spatially explicit.

Management strategies are associated with stages of the invasion process and are intended to strengthen the barriers that prevent a population from moving from one stage to the next (Fig. 2). For example, prevention measures are associated with the Introduction stage and may include phytosanitary treatments and border inspections, which form barriers to population introduction. In this review, we extend the prevention measures to include containment activities, which reinforce barriers to Reproduction and Dispersal of established populations. Surveillance and control (eradication) strategies are employed from the beginning of the Establishment stage to the end of the Spread stage, or Environment barrier, in order to find new populations of invaders. When they are discovered, newly established populations may be eradicated or contained to prevent or slow their spread. During the Spread stage, non-native populations readily occupy new sites. In this stage mitigation efforts, including control activities such as insecticide or biocontrol treatments, take place in an attempt to slow their spread and lessen their impacts.

While determining the optimal levels of prevention, surveillance, and control is a critical problem, the majority of OR applications focus on control strategies because biological invasions are often discovered when they are in the Spread stage, and control is needed to minimize their harmful impacts. Identifying optimal control strategies for established invasions is also important because optimal levels of prevention and surveillance investments depend on potential costs and damages, should prevention and surveillance strategies fail, and the invading population establishes and spreads. For example, if the expected costs and damages of a potential biological invasion are very large, then large investments in actions that prevent introduction, and find and eradicate established populations when they are small may be justified. Bioeconomic models of invasive species management have three features to minimize the total costs of damage and control (Epanchin-Niell and Hastings 2010): (1) invasion dynamics and the effects of control strategies on those dynamics, (2) damages caused by the invasion, and (3) costs associated with control. Because of the importance of determining optimal control strategies for established invasions, we review these three key components in the next sections.

3.1 Invasion dynamics

While life forms and stages of development may differ widely among invasive plant and animal populations, certain basic demographic processes are common to them all—birth, migration, aging, and death (Begon et al. 2009). These processes can be combined into a simple algebraic equation describing the change in population size:

$$N_{t+1} = N_t + B - D + I - E, \tag{1}$$

where N_t is the population size at time t, B is the number of new individuals born during period t, D is the number of individuals that die during period t, and I and E are the numbers of immigrants and emigrants, respectively, during the period. Individuals of all species pass through a number of life stages during their life cycle. For example, plants pass from seeds to seedlings to photosynthesizing adult plants. Further, seeds may remain dormant in the ground for many years before they germinate. To project plant population size, the demographer needs to estimate fecundity (average number of seeds produced per adult), seed germination rate (depending on the number of years of dormancy), seedling establishment rate, and adult survival rate. Therefore, Eq. (1) is only the basis upon which more realistic descriptions of population dynamics are built.

An invasion is initiated when humans introduce propagules of the invader to a novel environment. The chance of establishment depends on propagule pressure, a composite measure of the number of individuals introduced. Propagule pressure incorporates estimates of the absolute number of individuals involved in any one release event and the number of discrete release events. As the number of releases and/or number of individuals released increases, propagule pressure and the likelihood of establishment also increases.

The introduction of propagules will lead to a self-sustaining population only if the population is capable of growing, regardless of how often or how many propagules are introduced. The growth rate of the population depends on the fecundity and survival rates of individuals in different life stages. When the number of offspring that survive and grow into reproductive age classes repeatedly exceed the loss from those age classes due to mortality, the population will grow (Begon et al. 2009). In most populations, the fecundity and survival of individuals are adversely affected by intraspecific competition for mates and resources. Further, these adverse effects increase as the number of competitors increases. Then, the birth and survival rates of the population are density dependent, and this density dependence regulates population size. Intraspecific competition decreases the size of populations that are above a particular level but allow an increase in the size of the population below that level (Begon et al. 2009). This equilibrium level is called the carrying capacity of the population. In addition to intraspecific competition, birth and survival rates of a population are also affected by the environment (Gurevitch et al. 2011), and fluctuations in environmental conditions will cause fluctuations in population size away from the equilibrium carrying capacity.

For newly established populations of invasive species, there may be an inverse density dependence—the Allee effect—in which fecundity of the population is positively affected by increases in population size when population levels are very low (Taylor and Hastings 2005). This effect happens when mates have difficulty meeting and reproducing at low population levels, which may cause lower or negative population growth rates. As population size increases from these low levels, the fecundity rate increases thereby increasing population growth. At some point, population size reaches a level where intraspecific competition causes fecundity to drop, with associated negative effects on the population growth rate. Allee effects are important for understanding invasion dynamics because invasive species are often at low densities when introduced to an environment, and the effects on population growth may influence the effectiveness of control strategies (Liebhold and Bascompte 2003).

Whether populations produce sufficient numbers of offspring—new propagules—capable of dispersing to new areas will determine the potential for invasion spread. The theory of invasion spread began many decades ago with the use of reaction–diffusion models to fit dispersal data and forecast spread (Hastings et al. 2005). These models predict the population density across the landscape as a function of the population growth rate and a diffusion coefficient (rate of random movement across a homogeneous landscape). While this model has intuitive appeal, empirical evidence for a wide range of organisms has shown that dispersal is not random and in particular includes the presence of long-distance dispersal events that can greatly affect the rate of invasion spread. Further, because dispersal rates in many species depend on life stage, models that lack stage structure will overestimate invasion spread. Finally, Allee effects at the front of the invasion wave may also slow invasion spread. Perhaps the most limiting assumption of reaction diffusion models is a homogeneous landscape. The direction and distance of dispersing individuals as well as the survival and reproduction of those individuals is affected by landscape structure, which is the spatial distribution of resources affecting the stages of the invasion process (With 2002).

3.2 Invasion impacts on ecosystems and their services

Biological invasions may have wide ranging effects on the properties and processes of ecosystems (Simberloff et al. 2013). Ecological impacts can occur at the population, community, or ecosystem levels. A good example of an invasive species with a wide range of ecological impacts is the emerald ash borer (EAB), a phloem-feeding insect native to Asia and discovered in the U.S. in 2002 (Herms and McCullough 2014). Although the pathway and vector responsible for the EAB invasion remain unknown, EAB was probably imported into North America via crates, pallets, or dunnage made from infested ash in Asia. Since its discovery, EAB has spread to 26 states and two Canadian provinces, and threatens to extirpate native ash trees throughout North America. Given that ash is one of the most widely distributed tree genera in North America, the ecological impacts of the EAB invasion are likely to be experienced on a continental scale (Gandhi and Herms 2010a, b).

Biological invasions may also affect ecosystem services, which are the benefits that people derive from ecosystems (Millennium Ecosystem Assessment 2005). For example, ash trees on forest land may be harvested and sold as raw material for timber products. Ash trees in urban areas provide aesthetic amenities for urban dwellers. When ash trees succumb to EAB, the levels of these services (i.e., timber supply and aesthetic amenities) drop. Understanding the relative values of increases or decreases in different services can help managers select the management option that brings the greatest benefits to society. This requires the estimation and use of economic benefit functions, which quantify in monetary terms the relationships between changes in the provision of ecosystem services and changes in human well-being (Polasky and Segerson 2009; Champ et al. 2012).

3.3 Management activities and objectives

Prevention, surveillance, and control strategies may involve a wide variety of activities and associated costs. Prevention activities curtail immigration into a new location or limit the emigration from existing populations. Propagules of invasive species are often transported as hitchhikers on imported goods and prevention approaches may take place during any part of the commodity supply chain between offshore production and import. For example, in the horticulture industry, many species of plants are grown in offshore nurseries and then imported to the U.S., with the risk of transporting non-native insects and diseases. Pre-entry programs are a set of best management practices that offshore producers can implement to assure the production of high-quality goods that meet phytosanitary standards. Prior to or during transport, commodities may be treated with heat, cold, radiation, or pesticides to eradicate unwanted organisms. Goods and vessels may be inspected during transport or upon arrival in port to detect unwanted organisms. If unwanted organisms are found, shipments may require treatment and post-entry quarantine involving containment with a simultaneous inspection schedule. Prevention activities may also take place to limit the emigration of propagules from an established population to a new location. These activities include quarantines and barriers that limit spread. Surveillance activities take place in the importing country and involve actions to detect newly established pest populations. These actions may involve insect or animal traps, sentinel plots that are inspected for unwanted plants or pathogens, or visual surveys using air or ground transport to detect damage. Control strategies include reducing the size of the invasion by chemical, biological, mechanical, or manual control to slow or stop the spread of the invasive species. These activities may involve direct control of the pest itself or control of the host population. The costs of these activities vary and are usually available from government agencies or private contractors.

An objective that explicitly minimizes the costs of damage and management or maximizes the benefit of management subject to a budget constraint can be used to determine the economically optimal set of actions (Epanchin-Niell and Hastings 2010). For established invaders, the objective is to minimize the sum of control costs and invasion damages over time by choosing when, where, and how much to control. In many cases, the budget for control is limited, and then the problem is to minimize the cost of damage, subject to the budget constraint on the control activities. Optimal levels of prevention and surveillance activities depend on the expected costs and damages of the optimal control strategy, should prevention and surveillance strategies fail and the invading population establishes and spreads (Lockwood et al. 2005).

4 An optimization modeling framework for invasive species management

Mathematical models provide powerful conceptual frameworks to understand and predict biological invasion processes and design appropriate management strategies. Consequently, these models are extensively studied in invasive species management (Taylor and Hastings 2004). In this section, we present the ISM as a resource allocation problem, and describe the optimization model of Büyüktahtakın et al. (2015) to highlight various biological and economic aspects of an invasive species control problem. In particular, this spatially explicit optimization model includes biological growth dynamics (e.g., growth from seedbank, seed dispersal, and population transition among life-history stages), density dependence, and carrying capacity.

4.1 ISM: a spatio-temporal resource allocation problem

At its core, the invasive species management problem is a spatio-temporal resource allocation problem. Specifically, ISM can be defined as the problem of determining how to allocate limited resources for a set of management efforts (prevention, surveillance, and control) over space and time in order to optimize an economic and/or ecological objective. Consider an area consisting of I sites. Let x_i^t be a binary decision variable, which takes the value of 1 if a management action is applied on a site $i \in [1, I]$ in period $t \in [1, T]$, and 0, otherwise. Define p_i^t and c_i^t as the benefit (i.e., avoided damage) and cost of management action in site i and time period t, respectively. Then the spatio-temporal resource allocation problem can be formulated as

$$Max \left\{ \sum_i \sum_t p_i^t x_i^t \middle| \sum_i c_i^t x_i^t \leq B_t, \forall t \right\} \tag{2}$$

where B_t is the total resource budget at time period t. The objective function defines maximizing the value of investments over space and time. For example, in ISM, a decision-maker might wish to maximize the benefit of detection and eradication efforts within budget limitations over a planning horizon.

The resource allocation problem has been widely studied in the field of OR. This problem is equivalent to the classical knapsack problem (Nemhauser and Wolsey 1988a), which can be defined as one of selecting from a set of potential investments to maximize the sum of the benefits, subject to a budget constraint that cannot be exceeded. The knapsack problem is NP-hard (Kellerer et al. 2004). A range of integer and linear programming methods are available to solve problems of this form (Hillier and Lieberman 2012; Nemhauser and Wolsey 1988b).

4.2 An optimization model for controlling invasive species

We present below a comprehensive bio-economic model by Büyüktahtakın et al. (2015) for controlling invasive species to illustrate the main components of an OR model to help solve other similar control problems. The formulation of the MINLP model is described as follows: Let $t \in [0, T]$ be any year of the planning horizon, where T represents the final time period. The considered spatial location is divided into square sites with I rows and J columns. Any site of the location can be characterized by its coordinates (i, j), where $i \in \{1, 2, \ldots, I\}$ and $j \in \{1, 2, \ldots J\}$. Θ^{ij} is the set of neighboring sites of site (i, j). Age clusters (age categories) of the invasive species population are defined as $k = 1, 2, 3, \ldots, n+$, where k represents the age of each cluster, and $n+$ defines the age cluster n and older populations of the invader considered.

Other notation used in the model of Büyüktahtakın et al. (2015) is given below:

- E_{ijt}^k: The expected damage of the invader at site (i, j) at time t.
- P_{ijt}^k: The after-treatment population of the invader of age cluster k at site (i, j) at time t.
- D_{ijt}: The number of seeds dispersed from surrounding sites $(h, q) \in \Theta^{ij}$ to site (i, j).
- λ: The percentage of seeds that disperse from surrounding sites $(h, q) \in \Theta^{ij}$ to site (i, j).
- $\tau_{(h,q)}^{\theta}$: The probability of seed dispersal from surrounding sites $(h, q) \in \Theta^{ij}$ to site (i, j) with direction θ.
- S^k: The number of seeds produced by each stem of the plant in age cluster k.
- R_{ijt}: The number of seeds remaining in site (i, j) after dispersal to surrounding sites.
- ω: The percentage of seeds that remain after dispersal.

- B_{ijt}: The seed bank population in site (i, j) at time t, where B_{ij0} represents the initial seed bank
- γ: The longevity rate, i.e., survival rate of seeds in the seed bank.
- α: The germination rate of the seeds into a seedling.
- K_{ij}: Carrying capacity, which represents the maximum population that can be supported by site (i, j).
- $\tilde{P}^k_{ijt}$: The transition population of the invader in age cluster k without considering carrying capacity.
- ρ: The success rate of a seedling becoming a 1-year-old plant.
- ψ^k: The loss rate of individuals while transitioning from age cluster k to $k+1$.
- $\hat{P}^k_{ijt}$: The before-treatment population of invaders with considering carrying capacity.
- ϕ: The effectiveness rate of the control treatment (e.g., herbicide treatment).
- y_{ijt}: The binary variable, which is 1 if site (i, j) is treated in period t, and 0 otherwise.
- C_{ij}: Cost of treatment (e.g., labor and herbicide cost) per site (i, j).
- Ω: The available budget for the entire time horizon.

The MINLP model by Büyüktahtakın et al. (2015) is then formulated as follows:

$$\text{Minimize } Z = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T}\sum_{k=1}^{n^+} E^k_{ijt} P^k_{ijt} \tag{3}$$

subject to:

$$D_{ijt} = \sum_{k=1}^{n^+}\sum_{(h,q)\in\Theta^{ij}} \lambda\tau^{\theta}_{(h,q)} P^k_{hqt} S^k \quad \forall i, j, t \tag{4}$$

$$R_{ijt} = \sum_{k=1}^{n^+} \omega P^k_{ijt} S^k \quad \forall i, j, t \tag{5}$$

$$B_{ijt} = B_{ij0}(\gamma-\alpha)^t + \sum_{s=0}^{t}\left((\gamma-\alpha)^{t-s}(D_{ijs}+R_{ijs})\right) \quad \forall i, j, t \tag{6}$$

$$\tilde{P}^k_{ij,t+1} = \alpha\rho B_{ijt} \quad k=1; \forall i, j, t \tag{7}$$

$$\tilde{P}^k_{ij,t+1} = P^{k-1}_{ijt}(1-\psi^{k-1}) \quad k=2,\ldots,n-1; \forall i, j, t \tag{8}$$

$$\tilde{P}^k_{ij,t+1} = P^{k-1}_{ijt}(1-\psi^{k-1}) + P^k_{ijt}(1-\psi^k) \quad k=n^+; \forall i, j, t \tag{9}$$

$$\hat{P}^k_{ijt} = \min\left\{\left(K_{ij} - \sum_{v=k+1}^{n^+}\hat{P}^v_{ijt}\right), \tilde{P}^k_{ijt}\right\} \quad k=1,\ldots,n-1; \forall i, j, t \tag{10}$$

$$P^k_{ijt} = \hat{P}^k_{ijt}(1-\phi y_{ijt}) \quad \forall i, j, k, t \tag{11}$$

$$\sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{j=1}^{J} C_{ij} y_{ijt} \le \Omega \tag{12}$$

In this optimization model, the objective (3) minimizes the total expected damage resulting from the invasive species in all age clusters k, over all sites (i, j) and all time periods t. Equation (4) formulates seed dispersal to site (i, j) from surrounding sites $(h, q) \in \Theta^{ij}$. Equation (5) represents the number of seeds remaining at site (i, j) after dispersal to surrounding sites. Equation (6) represents the seed bank accumulation including the initial seed bank as well as all produced and dispersed seeds in site (i, j) from time zero until time t, while also considering the decay of seeds over time. According to Eq. (7), germinated seeds from the seed bank become seedlings turning into 1-year-old plants with a given success rate. Equation (8) implies that each individual moves up one age class in a 1-year period with a certain loss rate. Equation (9) represents the total population with maximum age cluster including all plants with age n+ and older until they die out. Equation (10) defines the carrying capacity constraints and gives the actual before-treatment population considering the maximum possible space left from older individuals of the invader population. In particular, Eq. (10) implies that the actual before-treatment population is assigned to the minimum of the remaining capacity left after older individuals populate a site (i, j), and the transition population. Equation (11) implies that the before-treatment population is reduced by ϕ percent if treatment is applied in site (i, j) at time t. Note that both Eqs. (10) and (11) are non-linear, however they could be linearized by conventional linearization methods (Büyüktahtakın 2017; Kibis and Büyüktahtakın 2017). Equation (12) limits the available treatment budget allocated for treatment throughout the entire time horizon.

The MINLP model proposed by Büyüktahtakın et al. (2015) is applied to control the infestation of Sericea (*Lespedeza cuneata*), an invasive plant threatening the Great Plains of U.S. However, their MINLP model provides a general framework for controlling invaders, and thus could also be adopted to other invasive species such as fish, insects, animals, and plants with age-specific vital rates. For example, equations representing age-structured growth (7)–(10) can be modified to formulate the growth of stage- or size-structured species, while seed dispersal and seed bank-based growth equations (4)–(6) can be adjusted to formulate various offspring generation and dispersal mechanisms. Furthermore, the age-specific carrying-capacity constraints (10) can be adjusted to bound the population size of species within its natural boundaries (Büyüktahtakın et al. 2015).

The intrinsic rate of growth, carrying capacity, form of invader's growth function, dispersal processes, and spatial representation are key factors to be considered in invasive species management planning. Most modeling studies on the ISM problem differ from each other according to how they model the interactions among these key factors. The growth of the invader population may be modeled using a linear, exponential, or logistic function, while also taking into account carrying capacity (the maximum population that can be carried in a specific location). For example, Hof et al. (1997) assume an exponential population growth, while a logistic growth is more realistic to represent the growth of many biological species. Büyüktahtakın et al. (2011a) propose a non-linear integer programming model by explicitly defining the logistic growth and carrying capacity of invasive species. Some studies further focus on biological details such as seedbank and age-structured growth and survival parameters as described in the MINLP model (3)–(12). In this MINLP model, Büyüktahtakın et al. (2015) formulate the population growth using a linear function, which defines the germination of seeds from the soil seed bank and dispersed seeds, as opposed to the use of a logistic growth function. These authors also incorporate non-linear carrying capacity limitations while giving priority to older-age clusters in the population in a spatially explicit model. They find that the predicted growth follows a multi-logistic population growth with multiple, sequential, and overlapping phases of logistic form (Meyer et al. 1999), rather than a simple logistic growth form.

Existing OR models usually consider the cost of management actions and a limited budget for treatment, while setting an objective that explicitly minimizes the cost due to invasion damages over time by determining when, where, and how much to control (Epanchin-Niell and Wilen 2012; Hof and Bevers 2000, 2002; Kaiser and Burnett 2010; Büyüktahtakın et al. 2011a; Horie et al. 2013). The cost of damages may include economic as well as environmental harm due to invaders. On the other hand, some studies focus on maximizing the social welfare and benefits from protected forestry services and products. Multiple objectives of the involved stakeholders have also been considered (see e.g., Büyüktahtakın et al. 2014a), while the analysis of such multi-objective optimization models (Kantas et al. 2015) has been limited in the ISM literature.

5 Classification of optimization models in ISM

In this section, we classify mathematical models with respect to the type of management action (prevention, surveillance, and control), biological complexity (e.g., growth, dispersal, and stage structure), and model complexity (budget constraints, spatial and temporal dimensions, and uncertainty), as shown later on in Table A. We also classify models with respect to their objectives and focus, the application considered, and main results, as shown later on in Table B.

5.1 Model classification based on stage of management

Figure 3 classifies the literature for ISM modeling with various types of complexities, and report the number of papers under each category and the corresponding percentage. The majority (86.7%) of reviewed mathematical modeling and optimization studies of ISM consider control programs either alone or in combination with prevention and surveillance activities (Fig. 3). Out of 60 modeling analyses, nine (15.0%) focus on prevention and control activities, ten (16.7%) focus on surveillance and control activities, and six (10.0%) focus on integrating prevention, surveillance, and control activities. Models that address prevention activities alone or in combination with other measures comprise 33.3% of the analyses, while models addressing surveillance cover 36.7% of the studies.

5.1.1 Control models

The majority of studies on invasive species management have concentrated on control strategies for established populations (Fig. 3a and Table A). Control strategies include reducing the size or slowing the spread of the invasion by chemical, biological, mechanical, or manual control, or other means (Olson 2006). By incorporating the effectiveness of the control method(s) selected, mathematical models [e.g., the MINLP model (3)–(12)] formulate control as a reduction in the size or rate of spread of the invading population. The optimization problem for control models is generally formulated to determine the optimal allocation of resources among control activities in order to minimize invasion damage over time while also satisfying a control budget and respecting biological dynamics of the invader (Table B). The resulting dynamic optimization problem can be addressed using various tools such as dynamic programming (DP), mathematical programming, and optimal control, see e.g., Billionnet (2013), Kibis and Büyüktahtakın (2017), and (Olson 2006).

5.1.2 Prevention models

Preventing the introduction and establishment of individuals of an invasive species is an important means of avoiding potential damages. Optimization studies of prevention address the efficient use of resources to curtail immigration into a new location or limit the emigration from existing populations. Propagules of invasive species are often transported as hitchhikers

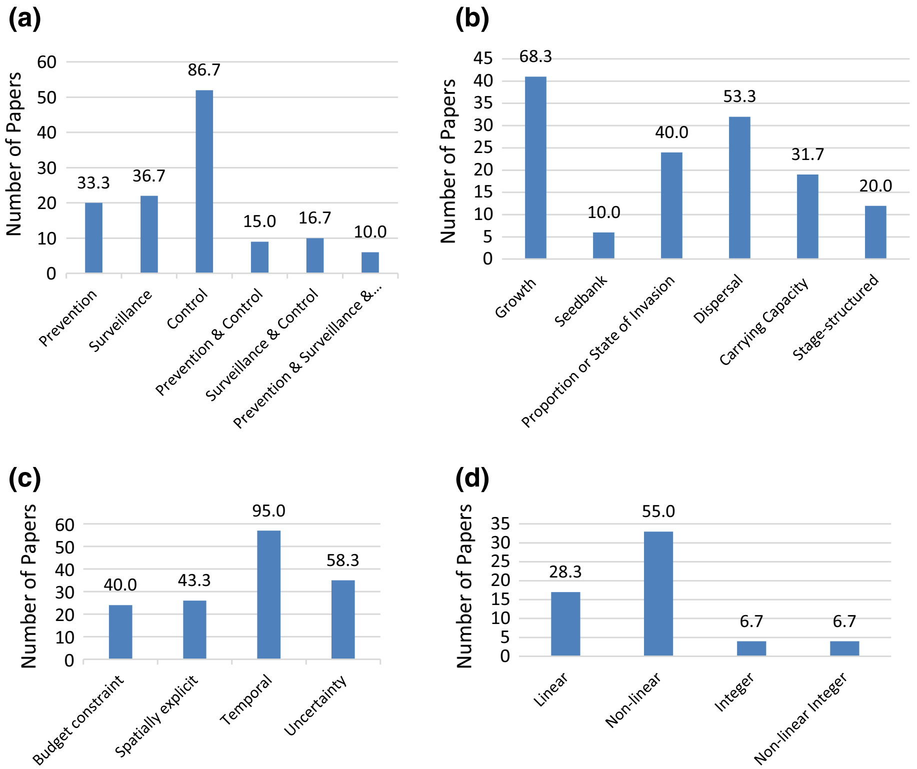


Fig. 3 Literature classification for ISM modeling in terms of the following: **a** stage of management, **b** biological complexity, **c** modeling complexity, and **d** mathematical complexity. Note that some studies are listed under multiple categories in each figure; therefore, the percentages do not total 100%

on imported goods, and the interception of infested material through border inspections is the primary means of curtailing immigration. Optimization models have focused on allocating an inspection budget among incoming shipments to minimize the introduction of infested shipments or infested units of an imported commodity into a novel environment (Chen et al. 2017; Surkov et al. 2009; Yamamura et al. 2016). These studies highlight the fact that optimal inspection policies focus more resources on the higher-risk commodities or pathways of introduction. Some recent mathematical models allocate resources to inspection and control activities during the introduction and establishment stages (e.g., Sanchirico et al. 2010), while other models design quarantine-restricted trade policies to prevent new introductions (e.g., Cook 2008). Optimization models have also accounted for the value of information from inspections to improve targeting of inspection resources over time in order to minimize accepted infested shipments (e.g., Springborn 2014). The problem of preventing biological invasions caused by ships transporting internationally traded goods between countries and continents is also studied from a long run perspective using queuing theory (Batabyal and Beladi 2006).

Prevention strategies also include quarantines and barriers that prevent the emigration of propagules from an established invasive population to new locations. Optimization studies analyze the efficient use of resources to establish barrier zones at the edge of the invasion front (Epanchin-Niell and Wilen 2012; Sharov and Liebhold 1998), contain or quarantine established populations (Moore et al. 2010; Pichancourt et al. 2012), and make sanitation cuts by preemptively removing the host population (Hof et al. 1997; Kovacs et al. 2014). One common goal in these models is to avoid damages and/or limit the costs of eliminating invasive species (Table B).

Several papers have studied prevention strategies jointly with control, and analyzed the related trade-offs for resource allocation (Burnett et al. 2008; Epanchin-Niell and Wilen 2012; Finnoff et al. 2007; Hof et al. 1997; Kovacs et al. 2014; Leung et al. 2002; Moore et al. 2011; Olson and Roy 2005; Pichancourt et al. 2012; Potapov 2009; Sharov and Liebhold 1998). A common result is that strategies for prevention, control, and damage reduction are complementary and the neglect of any of them may lead to unnecessarily large social costs (Gren 2008). These papers commonly report that investing in prevention activities is superior to investing in control strategies alone (Leung et al. 2002; Mehta et al. 2007). Leung et al. (2002) claim that a much higher monetary value should be placed on prevention than is currently done. Some researchers report that the allocation of resources among prevention and control depends on the related costs and effectiveness of the strategy (Kovacs et al. 2014; Potapov 2009). Potapov (2009) shows that the intensity of control depends on the trade-off between losses caused by invasion and the cost of control procedures. Kovacs et al. (2014) find that net benefits can be boosted when decision-makers cooperate in the allocation of resources among prevention and control actions. Olson and Roy (2005) show that an increase in the variability of introductions increases the marginal benefits from prevention and control for given levels of prevention and control. Sharov and Liebhold (1998) report that quarantine regulations may not always be an efficient means to reduce the rate of population spread, in particular when colonization rates are low.

5.1.3 Surveillance models

In addition to preventing the introduction of invasive species, optimal intervention strategies include surveillance to detect newly established populations. Both prevention and detection involve monitoring activities. With a prevention strategy, the purpose of monitoring is to prevent species from entering into an ecosystem. With a detection strategy, the purpose of monitoring is to determine the location and size of a population that already exists in the ecosystem. Detection strategies are commonly used with control in order to identify new establishments and quickly implement control measures for a prompt containment of invaders (Mehta et al. 2007; Homans and Horie 2011).

The costs of surveillance and control critically impact optimal intervention strategies. Control costs may be lower and eradication may be possible if a colonizing population is detected when it is small. However, the detection of a nascent population is often difficult, and the associated costs may be quite high (Mehta et al. 2007). In the surveillance-and-control problem, the manager seeks to identify the optimal search-and-treatment effort while minimizing damage from invasive species as well as the total cost of surveillance and control under a limited budget. These models provide a useful framework for determining the optimal timing and intensity of surveillance, assuming that treatment can be conducted only after the infestation is found (Bogich et al. 2008; Homans and Horie 2011; Horie et al. 2013; Mehta et al. 2007; Mbah and Gilligan 2010; Yemshanov et al. 2017). Resources for surveillance can also be allocated to maximize the expected number of transmission pathways that are covered by survey locations (Yemshanov et al. 2015).

5.1.4 Integrated prevention, surveillance, and control models

As noted above, control strategies focus on slowing the spread of an established population, prevention strategies limit the introduction of propagules of an invasive species, and surveillance increases the chance of early detection of newly established populations (Rejmánek and Pitcairn 2002; Rout et al. 2011). The optimal allocation of limited resources among these three different intervention activities is a critical research question in ISM. Greater spending on prevention could reduce the introduction of new invaders, while greater surveillance helps identify invasive subpopulations, which in return helps to control the spread of the species across the landscape. However, budget allocated to one of the three intervention strategies limits the budget allocated to the others. For example, while increasing surveillance may enable earlier control, the higher spending on surveillance could hinder eradication due to lack of funds. The relative costs and effectiveness of different interventions as well as their interactions affect the optimal timing and intensity of intervention strategies (Epanchin-Niell and Hastings 2010; Horie et al. 2013; Olson and Roy 2005). Fewer studies have examined both the optimal resource allocation and the timing of prevention, eradication, and control simultaneously, as opposed to studying these variables independently (Carrasco et al. 2010; Hyytiäinen et al. 2013; Polasky 2010; Rout et al. 2014, 2011; Mbah and Gilligan 2010).

The optimal investment of resources on each management action is sensitive to the unit costs and efficacy of different intervention measures, to the size of invader population after detecting invaders, and to the difference in the estimated impact of a localized and widespread invasion (Hyytiäinen et al. 2013; Rout et al. 2014). Furthermore, it is found that implementation of mixed strategies, where prevention, surveillance, and control are employed simultaneously, is usually better or more cost effective than implementing a single strategy alone when the extent of the infestation is uncertain (Rout et al. 2014).

5.1.5 Bio-economic model complexity

Simple invasion models are used to make general inferences about how systems work and offer analytical tractability by ignoring complex biological dynamics, environmental heterogeneity, or economic constraints. However, these models are often insufficient to realistically represent the spatio-temporal invasion patterns. On the other hand, each biological and economic factor considered adds a new level of complexity in mathematical models because these factors are generally represented as restrictions, increasing the number of variables and constraints in formulations. Thus, more detailed and complex models typically require detailed data and additional computational resources (e.g., CPU time and memory) to solve the model (Green et al. 2005). The trade-off between the realism of the model and its computational tractability makes the selection of the appropriate level of detail a challenge. Therefore, an adequate level of biological and economic complexity needs to be considered in order to forecast the dynamics of invasion processes and make the most appropriate management decisions to control invaders accordingly. Next, we will discuss various intricacies, including biological, spatial, and economical complexity as well as uncertainty investigated in previous studies.

5.1.6 Biological complexity

Earlier research highlights the sensitivity of the optimal management effort to biological and ecological factors such as intrinsic rate of population growth, carrying capacity, and the form of the invader's growth function, which are specific to species and site (Eiswerth and Johnson 2002; Kibis and Büyüktahtakın 2017).

Figure 3b presents a classification of ISM modeling in terms of the biological complexity. As can be seen, the majority of studies consider population growth (68.3%) by explicitly formulating the density of the species using growth functions, while others represent growth as a proportion of the landscape invaded or as a state of the invasion using a transition matrix representation (40.0%). Growth models can be categorized as density-independent and density-dependent. Density-independent models do not consider environmental limitations, such as the availability of essential resources (e.g., food and water), predation, disease, and thus carrying capacity (maximum population that a site can hold) is unlimited (Tilman 2004; Hairston et al. 1960; Menge and Sutherland 1987). These models typically assume a linear, exponential, or geometric population growth (Blackwood et al. 2010; Buhle et al. 2005; Finnoff et al. 2010; Taylor and Hastings 2004; Baxter et al. 2007).

Density-dependent models, such as the MINLP model (3)–(12), are more realistic because population growth rates are regulated by the density of a population. Models with density dependency include logistic growth (Albers et al. 2010; Büyüktahtakın et al. 2011a; Eiswerth and Johnson 2002; Leung et al. 2002), Gompertz function (Tjørve 2009), Allee effects (Burnett et al. 2008; Carrasco et al. 2010), and propagule pressure (Carrasco et al. 2010). Density-dependent models inherently include carrying capacity. While the majority of the studies formulate growth, less than half of them (31.7%) explicitly consider the carrying capacity limitations in their growth models.

Allee effects and propagule pressure have received limited attention in ISM modeling. One of the few exceptions is the work of Burnett et al. (2008), which assumes strong Allee effects and utilizes a minimum population threshold before which an invasive population of tree snakes cannot start growing in Hawaii. Carrasco et al. (2010) also incorporate Allee effects in a comprehensive bioeconomic model, while considering the exclusion, detection, and control of multiple invaders. They find that agencies should allocate less exclusion and more control resources to non-native invasive species characterized by Allee effects, a low generation rate of satellite colonies, and low propagule pressure.

Another implication of density dependence is the varying rates among the life stages of an organism (Tanner 1999). In particular, stage-structured models track the dynamics of a population partitioned into age, stage, size, and physiological classes; see, e.g., Getz and Haight (1989), Büyüktahtakın et al. (2015). Incorporating stage structure is important for cases in which vital rates such as growth, fecundity, and dispersal rates, and other critical parameters are age-, stage-, physiologically, or size-dependent and vary significantly among different life stages of a species (Caswell 2001; Cushing 1998; De Roos et al. 2003; Taylor and Hastings 2004). For example, Eqs. (6)–(8) represent the population transition among different stages (e.g., ages) of the species.

Taylor and Hastings (2004) consider a density-structured model in which a *Spartina alterniflora* population is classified by both local density (high and low) and age (juveniles and adults). Other ISM models are structured by age (Haight and Polasky 2010; Hyytiäinen et al. 2013; Leung et al. 2002; Büyüktahtakın et al. 2015), colony age (Sharov and Liebhold 1998; Epanchin-Niell et al. 2012), physiological stage (Cacho et al. 2007), life stages (Pichancourt et al. 2012), and individual size (Epanchin-Niell et al. 2014) of an invasive species.

The main advantage of a structured model is that it allows using different vital rates for different classes (Taylor and Hastings 2004). Stage structures are typically represented using matrices, which define transitions in the life cycle of organisms. These transitions and stage-specific parameters can provide insights into management strategies regarding which stage should be given priority to target in order to maximize the management impact on the population (Caplat et al. 2012; Ramula et al. 2008; Büyüktahtakın et al. 2015).

Few studies (10.0%) explicitly consider the seedbank or offsprings in their mathematical models. Most of them formulate state transitions from the seedbank stage to the adult stage using matrix representations (Baxter et al. 2007; Cacho et al. 2007; Firn et al. 2008; Pichancourt et al. 2012; Büyüktahtakın et al. 2015).

5.1.7 Spatial optimization and dispersal processes

Capturing spatial considerations in ISM is essential because a great part of an ecosystem's structure, function, and processes is spatial in nature (Berec 2002; Durrett and Levin 1994; Hof and Bevers 1998). Such processes include offspring dispersal and establishment, movement among regions, local competition, and impacts of spatial heterogeneity (e.g., soil fertility, environmental, and other abiotic factors). In order to formulate heterogeneous growth and spread over space, one needs to use spatial models. Figure 3c presents a classification of ISM modeling in terms of the modeling complexity with respect to considering spatial and temporal dimensions, economic constraints, and uncertainty.

A spatially explicit mathematical model represents a continuous or discrete heterogeneous space in which the variables, inputs, or processes have explicit spatial locations (Scheller and Mladenoff 2007). Here we summarize three common approaches used to incorporate the space dimension into ISM models, similar to the approach of Gilligan and van den Bosch (2008). These involve the spread of invasive populations over a landscape using the following geometries:

- Regular-shaped gridded landscapes divided into rectangular, square, triangular, or hexagonal cells, which have been widely used in dynamic spatial modeling and the study of ecosystems including invasive species (Billionnet 2013; Büyüktahtakın et al. 2011a; Epanchin-Niell and Wilen 2012; Hof and Bevers 1998, 2002; Potapov and Lewis 2008; Tyre et al. 1998; Aadland et al. 2015). For example, the MINLP model (3)–(12) represents a region, which is divided into equal-sized square grid cells, each representing a site of the landscape considered. Grid representation is particularly used to model the nearest neighborhood interactions, e.g., spread of offspring to adjacent neighbors, such as in Eq. (4). In a more realistic representation, irregular polygons are used to represent land parcels (Billionnet 2013). However, the rectangular grid with regular-shaped cells is generally preferred in modeling due to its symmetrical, orthogonal coordinate system and the typical use of rasters from Geographic Information Systems (Birch et al. 2007).
- Networks that represent short- and long-distance movement, e.g., spread of offspring and migration of invasive individuals, and irregular long-distance movement through animal and human movement.
- Diffusion processes, which represent large-scale dispersal across a landscape according to a dispersal kernel.

Although considering the spatial dimension is critical in ISM modeling, less than half of the reviewed papers focus on the representation of space in a mathematical optimization model (Hof et al. 1997; Sharov and Liebhold 1998). This is because spatio-temporal characteristics significantly increase the complexity of these models, making the solution often computationally intractable (Holst et al. 2007; Pacala and Silander 1990). Integrating spatial dimensions also requires the use of mathematical programming, for which closed-form solutions are typically not available.

Incorporating spread processes into management models is an essential factor for determining informed management strategies (Caplat et al. 2012). Despite this fact, dispersal of invasive species has only been modeled in slightly more than half (53.3%) of the studies considered. Reaction–diffusion (Skellam 1951; Fisher 1937) and integro-difference (Neubert and Caswell 2000; Kot and Schaffer 1986) models are commonly used approaches for defining the spread of invasive species in an optimization model. Reaction–diffusion models are based on partial differential equations (PDEs) and neglect individual behavior, thus are more suitable for cases that do not require much individual detail. Unlike reaction–diffusion PDEs, which assume reproduction and dispersal occur simultaneously, integro-difference equations break dispersal and population dynamics into separate stages, and define spread using a dispersal kernel (Kibis and Büyüktahtakın 2014). Another common approach in ISM is to model the spread of offspring to adjacent neighbors using a grid representation of adjacent cells, while incorporating cell-to-cell dispersal probabilities that decline with distance and are based on empirical observations; see e.g., Büyüktahtakın et al. (2011a), Hof and Bevers (1998), Getz and Haight (1989). In addition to these methods, the gravity method (Potapov and Lewis 2008), individual-based model (DeAngelis and Mooij 2005), and network model (Chadès et al. 2011; Yakob et al. 2008) are used to formally integrate spatial heterogeneity and spread into mathematical models. For example, Potapov and Lewis (2008) model dispersal between lakes using a gravity model, which employs the attractiveness of and distance to a location for forecasting travel patterns. While individual-based models focus on tracking the dispersal behavior of individuals or groups of similar individual organisms (Grimm and Railsback 2013), network models represent the movement over a set of neighboring vertices of a network graph through links connecting those vertices (Chadès et al. 2011).

5.1.8 Economic constraints and uncertainty

The budget allocated for any intervention strategy is usually limited in ISM. Budget constraints [see, e.g. Eq. (12)] are also shown to substantially impact the optimal management strategy compared to unconstrained optimization (Taylor and Hastings 2004). However, only 40.0% of mathematical modeling studies explicitly consider the budget constraint in their model. This may be due to complications that it causes, which make it difficult to obtain closed-form solutions. Some of the models without budget constraints determine optimal actions to minimize cost of management actions as well as loss due to invaders. In those models, the budget is determined optimally in the objective, while not considering a prior budget restriction. On the other hand, we observe that most studies (95.0%) consider the temporal dimension of the problem, while spatially explicit models form only 43.3% of the total.

Predicting and controlling biological invasions is a highly difficult problem because of inherent uncertainties. Examples of those uncertainties include unpredictable weather conditions and environmental disturbances, natural stochasticity (e.g., initial seed bank, changing fecundity and mortality, random dispersal), varying management practices, and uncertainties associated with estimating parameters of population growth and spread.

The uncertainty about the behavior of invasive pest populations is a fundamental challenge for invasive species managers. Slightly more than half of the studies (58.3%) explicitly consider uncertainty in one or more model parameters. For example, the following have been modeled and the relative impacts studied: uncertainty in dispersal (Cacho et al. 2010; Hyder et al. 2008; Kovacs et al. 2014; Kibis and Büyüktahtakın 2017), current extent of the invasion (Moore et al. 2011; Rout et al. 2014), probability of introductions (Olson and Roy 2005) and establishments (Carrasco et al. 2010), probability of infection or invasion

(Hyytiäinen et al. 2013; Potapov 2009; Sebert-Cuvillier et al. 2008; Haight et al. 2011) probability distributions of the degree of infestation (Haight and Polasky 2010; Horie et al. 2013), probability of species presence (Hauser and McCarthy 2009; Regan et al. 2006), probability of disturbance (Firn et al. 2008), probability of connection among nodes of a network (Yakob et al. 2008), probability of detection (Baxter and Possingham 2011; Bogich et al. 2008; Cacho et al. 2007; Epanchin-Niell et al. 2014; Hauser and McCarthy 2009; Horie et al. 2013; Mehta et al. 2007; Moore and McCarthy 2016; Polasky 2010; Regan et al. 2006), state-transition probabilities (Bogich and Shea 2008; Epanchin-Niell et al. 2012; Finnoff et al. 2007; Leung et al. 2002; Moore et al. 2010; Potapov 2009; Rout et al. 2014, 2011), probability of successful eradication (Green et al. 2005; Rout et al. 2014, 2011), and probability of containment (Moore et al. 2011). While most ISM models that account for uncertainty assume that decision makers are risk neutral, a few models assume that decision makers are risk averse, which affects the optimal allocation of resources (Finnoff et al. 2007; Springborn 2014). For example, assuming risk neutrality, Leung et al. (2002) show that invasive species are managed more cost effectively when greater investments are allocated to prevention activities relative to control. However, assuming risk aversion, Finnoff et al. (2007) show that greater investments in control activities are preferred when the outcomes of controls are more certain than prevention activities. Incorporating uncertainty further complicates the mathematical model, necessitating the use of stochastic modeling and advanced solution algorithms. We will discuss these modeling and solution techniques in detail in Sect. 5.

5.1.9 Spatial and temporal size

Table A also classifies models with respect to their spatial and temporal size. Most ISM studies consider a multi-period problem, while 56.7% of them only consider one region, i.e., are not spatially explicit. Depending on the complexity of the model and the selected solution method, we observe that the considered spatial and temporal sizes largely vary among different studies. For example, spatial size varies from four regions to 2000×2000 grid cells (4,000,000 regions), while temporal size changes from $T = 4$ periods to $T = 200$ periods. Each period is usually defined as a year in ISM models.

5.1.10 Mathematical complexity

Depending on the linear or non-linear nature of the objective function and the constraints in an ISM model, we categorize the models as linear, non-linear, integer, and non-linear integer (Fig. 3d). The majority (55.0%) of the models in ISM optimization are non-linear, while 28.3% of them are linear. Out of 60 modeling analyses, only four (6.7%) are linear-integer models including discrete variables, while four (6.7%) are non-linear integer models. Section 5.3 presents various solution approaches to deal with linear, integer, and non-linear models.

5.2 Classification based on focus, objective, and application

Table B presents a classification of models with respect to their objectives and focus, the application considered, and main results. As can be seen, the majority of studies focus on invasive plants (21) followed by pest species (19), while seven of them focus on invasive animal species, six of them study aquatic invaders, and seven of them consider a hypothetical species. As shown in Table B, a typical study on ISM modeling and optimization focuses on a single invading terrestrial plant or pest species in Europe, North America, or Australia. We also report on a few studies that were performed in New Zealand, South Africa, and the Baltic Sea.

The ISM problem can be defined with an objective of minimizing the cost of damages due to invaders over time with respect to a management budget (e.g., Büyüktahtakın et al. 2015; Eiswerth and Johnson 2002; Kibis and Büyüktahtakın 2017), while determining the time, location, and intensity of control. Other objectives include minimizing the economic damages caused by invasive species and the cost of management actions rather than using a treatment budget (e.g., Homans and Horie 2011; Kaiser and Burnett 2010; Epanchin-Niell et al. 2012; Olson and Roy 2005; Carrasco et al. 2010), minimizing economic and ecological costs due to invaders (e.g., Bhat et al. 1993; Burnett et al. 2008), minimizing the total invasive population (e.g., Bogich and Shea 2008; Hof and Bevers 2000), and minimizing the number of newly infected populations (Horie et al. 2013). Other studies choose to maximize the social welfare and net benefits from protected forestry services and products (e.g., Albers et al. 2010; Aadland et al. 2015; Finnoff et al. 2010; Leung et al. 2002; Kovacs et al. 2014), maximize the probability of detection or expected number of detections (e.g., Demon et al. 2011; Moore and McCarthy 2016), and maximize the number of healthy individuals (e.g., Baxter et al. 2007; Cacho et al. 2007; Firn et al. 2008; Büyüktahtakın et al. 2015; Mbah and Gilligan 2010). Multiple objectives of the involved stakeholders, such as damage to three impacted resources—saguaros, buildings, and vegetation—are also considered in the optimization models (Büyüktahtakın et al. 2011b, 2013, 2014a).

The specific conclusions and policy recommendations made by the studies depend on the focus of the study and assumptions made in the model. For example, the initial extent of invasion, consideration of spatial factors, the studied species, and thus growth and dispersal rates are all different. All proposed results could be valid, because one policy does not fit for all situations due to the complexity of the problem.

One recommendation for the effective control of invasive species is the early removal of infested populations (Blackwood et al. 2010) and increased trapping strategy in the initial years of invasion (Bhat et al. 1993). Consecutive treatment is recommended for Buffelgrass invasion in Arizona (Büyüktahtakın et al. 2011a), while another study suggests that every 2–3 year treatment may be effective for Sericea Lespedeza due to the reproductive maturity (Büyüktahtakın et al. 2015). While some studies suggest that control should focus on highly invaded areas (Finnoff et al. 2010), some studies recommend targeting new pockets for treatment (Haight et al. 2011) and smaller size classes than adults (Pichancourt et al. 2012). On the other hand, Taylor and Hastings (2004) suggest that optimal control strategy depends on the available budget. The authors report that low-density areas should be targeted under low budget, while the treatment should focus on high-density areas when the budget is high. Yemshanov et al. (2017) suggest that it is optimal to spend approximately one-fifth of the budget on surveys and the rest on tree removal, while focusing on the sites with the highest probabilities of pest introduction.

Optimal treatment also depends on the area, density, and planning horizon (Hyder et al. 2008), while size of the initial introduction, spatial heterogeneity, and natural disturbances could impact the invasion processes (Sebert-Cuvillier et al. 2008), and thus optimal treatment. The success of eradication also depends on the detectability of the target plant, the effectiveness of control, labor requirements for search and control, and the germination rate of the plant (Cacho et al. 2007).

Büyüktahtakın et al. (2014a), Grimsrud et al. (2008), and Kovacs et al. (2014) are among researchers who advocate coordinated efforts among stakeholders (e.g., managers, ecologists, government) involved in invasive species management to minimize harmful impact or maximize net benefits. The multi-objective approach of Büyüktahtakın et al. (2014a) highlights that the cooperation of different interest groups is absolutely necessary in establishing acceptable treatment strategies; otherwise, the total damage due to the Buffelgrass invasion in Arizona becomes very large. These coordinated efforts include the compensation of one or more stakeholders for achieving an agreed optimal solution that leads to the most effective use of shared resources. For example, using a cooperative game-theoretic model, Büyüktahtakın et al. (2013) specifies that a homeowner strategy of protecting against wildfire risks due to Buffelgrass invasion in Arizona affords less protection to other resources such as saguaros and riparian vegetation that are also impacted by Buffelgrass invasion. A similar result holds for protecting saguaros, which are also spatially concentrated. Under the optimal solution, groups caring about riparian vegetation would compensate homeowners and groups caring about saguaros because both of these groups' gains are reduced by agreeing a cooperation with the groups caring about riparian vegetation.

Optimal detection depends on the ease of detection, growth parameters, and initial population size (Mehta et al. 2007). Optimal surveillance effort depends non-monotonically on population growth rate (Bogich et al. 2008; Epanchin-Niell et al. 2012). Optimal sample densities are lowest for populations with very low growth rates (and long time horizons for detection), because they remain small longer and are less expensive to eradicate. Optimal surveillance effort is highest for infestations with moderate growth rates because their damage and eradication costs grow quickly but they may not be easy to detect. Species with very high growth rates warrant less surveillance effort because they are easier to detect. Baxter and Possingham (2011) suggest focused searching if the invasion is not widespread, while Horie et al. (2013) suggest focused searching on sites with a high expected number of infested trees. According to Hauser and McCarthy (2009), surveillance should be prioritized in environments where detection is easy, and a moderate investment is necessary to ensure a high probability of detection.

Leung et al. (2002) suggest that a much higher value should be placed on prevention, while Burnett et al. (2008) suggest allocating more funds to detecting small-size populations rather than prevention efforts. Sharov and Liebhold (1998) also claim that quarantine regulations may not always be an efficient means to reduce the rate of population spread. According to Moore et al. (2010), quarantine is optimal if the pest leads to huge costs in terms of damage and treatment, while surveillance is optimal if it is more cost effective than quarantine. Rout et al. (2014) report that it is optimal to combine surveillance efforts with quarantine and control. Optimal management depends on the effectiveness of each action and different stages of invasion. If the pest is absent, then it is more effective to prevent impacts through prevention and surveillance (Rout et al. 2011). On the other hand, an intermediate level of detection is optimal, according to the study of Mbah and Gilligan (2010).

5.3 Classification based on solution approaches

In this section, we discuss various modeling and solution approaches in ISM, including optimal control, stochastic dynamic programming, the partially observable Markov decision process (POMDP), simulation, numerical analysis, mathematical programming [linear programming, mixed-integer programming (MIP), and mixed-integer non-linear programming (MINLP)], heuristics, and other methods. Figure 4 shows the number and percentage of studies with respect to different mathematical modeling and the solution approaches. As can be seen, the majority (19.8%) of the studies in ISM use optimal control, while 18.5% use SDP and 2.6% use POMDP approaches. Simulation and numerical analysis forms 21.0% in total,

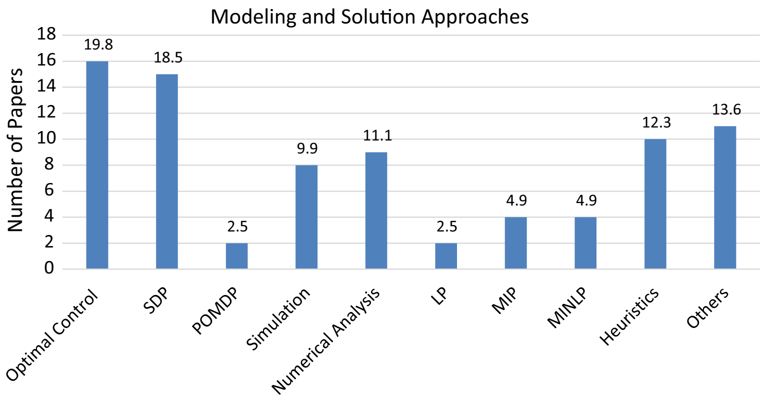


Fig. 4 Classification of papers according to modeling and solution approaches, with related percentage distribution on top of each bar

which is followed by heuristics (12.3%) and other methods (13.6%). We also observe that MP modeling approaches (LP, MIP, and MINLP) form 12.3% in total.

Researchers use various approaches to solve mathematical problems in ISM, as described below.

5.3.1 Optimal control and numerical analysis

A common approach to capture non-linearity in growth dynamics applies optimal control to an ISM model. Optimal control models, similar to OR models, explicitly state the management objective, and alternative actions or controls (Baker and Bode 2013). In an optimal control model, the growth and spread of the invasive species is typically represented by a biologically determined transition equation, such as the growth from seedbank, seed dispersal, and population transition given in Eqs. (4)–(10), while the state of invasion is defined by the size of the biomass or population of the invader, see e.g., Billionnet (2013), Hof and Bevers (1998). Control action(s) involve(s) reducing the size of the invasion by chemical, biological, mechanical, manual, or other means, or prevention and surveillance measures. The manager's objective is to minimize expected discounted control costs and invasion damages over time, subject to the biological transition function for the invasion (Olson 2006). One common approach to solving this dynamic optimization problem is optimal control. Such an approach has been applied to control and removal programs (Albers et al. 2010; Baker and Bode 2013; Blackwood et al. 2010), surveillance programs (Hauser and McCarthy 2009; Homans and Horie 2011; Mehta et al. 2007; Grimsrud et al. 2008), and prevention programs (Burnett et al. 2008; Kovacs et al. 2014; Olson and Roy 2005; Mbah and Gilligan 2010).

Nonlinearity in the system of equations hinders an analytic solution in an optimal control model. In order to facilitate a solution, numerical analysis methods have usually been employed to present economic and biological insights in optimal control and other analytical models (Bogich et al. 2008; Cacho et al. 2007; Epanchin-Niell et al. 2014; Moore et al. 2011; Sharov and Liebhold 1998). In particular, researchers use ordinary differential equations (Albers et al. 2010; Finnoff et al. 2010; Homans and Horie 2011; Grimsrud et al. 2008), necessary and sufficient optimality conditions such as Pontryagin's maximum principle and the Hamilton–Jacobi–Bellman equation (Burnett et al. 2008; Hauser and McCarthy 2009;

Kovacs et al. 2014; Mehta et al. 2007; Olson and Roy 2005), linear-quadratic control (Blackwood et al. 2010), and numerical solution methods, such as the boundary value problem and Hamiltonian (Burnett et al. 2008; Mbah and Gilligan 2010).

Optimal control software in MATLAB has been commonly used by researchers to tackle optimal control models. The optimal control problem can also be represented in a discrete form using the MINLP formulation (3)–(12) of Büyüktahtakın et al. (2015), which could be directly solved by non-linear optimization software such as SNOPT, CONOPT, and BARON. Kibis and Büyüktahtakın (2017) provide an equivalent linear formulation (MIP) for the MINLP (3)–(12), which could be solved using commercial software such as CPLEX and Gurobi.

5.3.2 Equilibrium and partial equilibrium analysis

Some of the earlier work has concentrated on the long-term state of the invasion rather than the short-term invasion dynamics. Equilibrium analysis usually assumes a time horizon sufficiently long for reaching equilibrium and minimizes some function of the equilibrium state of the invasion. A typical equilibrium analysis determines the minimum level of control or surveillance such that the invasion is eradicated in the long-term (Epanchin-Niell et al. 2012). That work defines the long-term epidemic equilibrium, either in terms of determining the optimal long-term equilibrium surveillance effort (sample density that minimizes the total expected costs of surveillance, eradication, and damages over time) (Epanchin-Niell et al. 2012) and expected equilibrium number of populations in each size class, or stock sizes (Eiswerth and Johnson 2002). Eiswerth and Johnson (2002) show that in equilibrium, the benefits and costs of invasive species management must be equal at the margin.

Partial equilibrium (PE) models have been used to compute the long run economic consequences of non-native pests on an agricultural market, while assuming that other markets remain unaffected by supply and demand shocks. In particular, PE analysis has been performed to control the introductions of non-native harmful pests in agriculture via changing import and trade policies and analyze the tradeoff between changes in trade policies and damages caused by the invader's introduction with a particular trade policy (Breukers et al. 2008; Cook 2008; Surkov et al. 2009). For example, Cook (2008) assess the likely regional economic welfare implications of a new Australian biosecurity regulation that could also increase the risks of potential pest invaders. On the other hand, Surkov et al. (2009) use a PE analysis to determine the optimal inspection policy in the Netherlands given the estimated costs of introduction of pests through trade pathways, while accounting for the potential price effects due to the pest's introduction. Breukers et al. (2008) analyze the impacts of repeated brown rot outbreaks on supply and demand of seed potatoes using a PE model.

5.3.3 Stochastic dynamic programming and Markov decision processes

Dynamic programming, a method first introduced by Bellman (1957) is a procedure for optimizing multistage decision processes. DP effectively decomposes highly complex problems into a series of sub-problems which are solved recursively (Büyüktahtakin 2011).

The growth transition equations can be considered as a Markov decision process (MDP). Stochastic dynamic programming is a rigorous tool that can be used to solve MDPs in order to find the optimal state-dependent decisions for a stochastic system (Bellman 1957;

Kennedy 1981; Mangel and Clark 1988). SDP is used extensively in the optimization of invasive species control (Finnoff et al. 2007; Leung et al. 2002; Moore et al. 2010; Rout et al. 2011; Rabbinge and Rossing 1987; Yokomizo et al. 2009), surveillance (Baxter and Possingham 2011; Pichancourt et al. 2012; Regan et al. 2006), and prevention (Finnoff et al. 2007; Leung et al. 2002), and the combination of prevention, control, and surveillance (Hyytiäinen et al. 2013; Moore et al. 2010; Polasky 2010; Rout et al. 2011). The majority of studies incorporating uncertainty also use the SDP approach. SDP has been quite popular and successful in ISM because the nonlinear and stochastic features that characterize a large number of ecological systems can be translated into an SDP formulation.

One particular challenge of SDP is the uncertainty about the state of the system when applying state-dependent decision models to ISM problems. The partially observable MDPs can be applied to problems where the decision-maker does not know the state of the system but makes observations that are probabilistically linked to this underlying state (Littman 2009; Monahan 1982). POMDPs have been successfully used in invasive species surveillance and control (Haight and Polasky 2010) and prevention (Rout et al. 2014).

5.3.4 Simulation methods and numerical procedures

Simulation is a modeling technique that is used to imitate the behavior of a real-world system using a computer program, representing all the characteristics of the system by a mathematical or algebraic description (Ackoff 1961; Maass et al. 1962; Yeh 1985). Simulation models have been successfully used in ISM to simulate the growth and spread of an invasive population in a heterogeneous landscape and study the impacts of management actions (Baxter and Possingham 2011; Cacho et al. 2010; Carrasco et al. 2010; Haight et al. 2011; Sebert-Cuvillier et al. 2008).

For example, by changing several parameters in the simulation, Sebert-Cuvillier et al. (2008) examine various hypotheses regarding the role of a number of mechanisms on invasion dynamics, such as spatial heterogeneity, seed dispersers, site of first introduction, large-scale natural disturbances, and forest management. Using simulation under different assumptions, Baxter and Possingham (2011) compare the relative performances of SDP recommendations and alternative management strategies. On the other hand, Yakob et al. (2008) utilize stochastic network simulations to study population heterogeneity as a function of landscape structure and invader behavior. Other simulation models evaluate alternative sampling designs (random placements, grid arrays, transects, sectors and annuli arrangements) and a common kernel (Skarpaas et al. 2005) as well as the spatial arrangement of sample points and the overall cost of detecting and eradicating invasive populations (Berec et al. 2015).

Mathematical programming methods maximize or minimize an objective to find the optimum decision for system operation while meeting all system constraints. On the other hand, a simulation model provides the response of the system for certain inputs, i.e., it helps a decision-maker to examine the consequence of various decision rules or scenarios of a system. In particular, optimization considers all possible decision alternatives, while simulation analyzes only a limited number of input decision alternatives. However, the advantage of simulation relative to MP is that it is more flexible and adaptable in simulating the response of the system.

Recently, an optimization scheme is incorporated into simulation in order to perform certain degrees of optimization. For example, Carrasco et al. (2010) use genetic algorithms combined with Monte Carlo simulation in a simulation-optimization routine in order to minimize the net present value of total costs due to invasions and their exclusion, detection, and control. Using a network simulation approach combined with an MDP process, Chadès et al. (2011) derive general rules for managing and surveying networks of pests, diseases, and endangered species.

5.3.5 *Mathematical programming (linear, integer, non-linear, and non-linear integer) approaches*

As discussed previously, simulation approaches have been widely used to represent and provide insight into ecological functions. However, optimization approaches can offer a powerful alternative to simulation because mathematical optimization can evaluate very large numbers of treatment alternatives and permit trade-off analyses that might otherwise be impossible (Hof and Bevers 2002). Another advantage of mathematical programming models compared to simulation and statistical models is their flexibility for modification and their generalizability to different types of applications.

Most optimization models are based on some type of mathematical programming technique such as linear programming, integer programming, non-linear programming, and non-linear integer programming. An extensive literature review of MP approaches in ISM reveals that no general MP model or algorithm exists for addressing ISM decision problems. The choice of MP depends on the characteristics and invasion dynamics of the invasive species studied, biological and economic complexity handled, and specific objectives and constraints.

Among the first MP studies in spatial ISM are the studies of Hof et al. (1997) and Hof (1998) in which an LP model is developed in order to control the growth and dispersal of an exotic pest on a gridded landscape structure. Different from the work of Hof (1998), Hof et al. (1997) incorporate the carrying capacity limitation and propose an integer programming formulation in order to control a forest pest. Hof and Bevers (2000) and Hof and Bevers (2002) present various spatio-temporal optimization models, in particular LP approaches for ecological systems management.

A logistic growth is more realistic than an exponential growth to represent the biological growth dynamics of the species. Büyüktahtakın et al. (2011a) propose a non-linear integer programming model by explicitly defining the logistic growth and carrying capacity of invasive species. The MINLP (3)–(12) model of (Büyüktahtakın et al. 2015) explicitly formulates growth from the seedbank while taking into account age-structured carrying capacity. Due to the complexity of the proposed MINLP, the authors use a rolling horizon approach to solve the problem.

The computational complexity of MINLP models has remained the biggest challenge to their implementation in computational software. Kibis and Büyüktahtakın (2017) address this computational challenge by presenting an MIP model that provides an exact optimal solution contrary to the earlier MINLP model proposed by Büyüktahtakın et al. (2015). The authors first formulate an MINLP model that integrates integer treatment variables and dispersal probabilities into a spatially explicit age-structured model, and then they linearize the MINLP, which results in an equivalent MIP that can be more efficiently solved. Their results show that the proposed MIP model outperforms the equivalent MINLP and non-linear programming (NLP) [MINLP relaxation] models in terms of solution quality and potential problem size that could be tackled.

Büyüktahtakın et al. (2014a) propose a multi-objective MINLP formulation to simultaneously optimize three objectives of minimizing damages corresponding to three different threatened resources by Buffelgrass invasion in Arizona: saguaros (a native cactus species), buildings and vegetation. The authors use a multi-objective distance-based approach (Szidarovszky et al. 1986) to determine an overall optimal solution, which has the minimum distance to the ideal solution that optimizes each single objective separately. The distance measure is determined based on the assumption of the compensation between the objectives. For example, while under full compensation Manhattan distance is used, under partial compensation Euclidean distance is computed. When objectives are normalized, the total damage values do not show much variability in comparing different distances.

Other MP approaches are proposed with different assumptions and focus on invasion dynamics. For example, Epanchin-Niell and Wilen (2012) study a spatially explicit MIP model, in which they consider the presence or absence of propagules (e.g., seeds or spores) on a cell rather than the abundance of propagules per cell. Hastings et al. (2006) develop simple approaches based on LP for determining the optimal control strategies of different stage or age classes of invasive species that are still in a density-independent phase of growth. Later, Horie et al. (2013) propose a scenario-based MIP model to optimally survey and treat an invasion while approximating uncertainty about the extent of the infestation in each site by a set of scenarios.

5.3.6 Heuristic methods

Many studies in the literature develop heuristic algorithms to solve ISM decision problems because the optimal solution is typically computationally intractable. Heuristic methodologies do not guarantee an optimal solution as in exact solution approaches. However, they can provide a close-enough solution for the specified objective function in a relatively short amount of computational time, and their use can be easier for practitioners. In particular, for large-size problems in ISM, heuristic methods and meta-heuristic algorithms such as the genetic algorithm (Kaiser and Burnett 2010; Taylor and Hastings 2004), simulated annealing (Demon et al. 2011), and neural networks (Potapov 2009) are utilized by researchers.

In order to handle the computational difficulty of mathematical programming models in ISM, some researchers use a rolling horizon heuristic, where the nonlinear programming model is solved for each period, and the resulting population after management actions are employed is used as the next period's initial condition (Büyüktahtakin et al. 2014b; Aadland et al. 2015). While this method may lead to quick solutions, it may lead to suboptimality, compared to solving the full dynamic model, which handles current and forecasted damages at the same time.

Both heuristic and exact solution methods have their own advantages and disadvantages. For instance, when facing a large-scale complex problem, heuristic and metaheuristic algorithms can be used to find acceptable but non-optimal solutions. On the other hand, analytical and exact methods usually guarantee optimality, while they are rarely applicable to real-sized non-linear instances. In such cases, approximation or hybrid methods arise as an acceptable way to solve complex problems practically (Govindan et al. 2015). Some examples of the hybrid approaches used in ISM include 2–factor approximation (Moore and McCarthy 2016), first-order approximation, and neuro-dynamic programming (Potapov 2009). Nicol and Chadès (2011) also develop a heuristic sampling method that approximates the optimal policy for any starting state in order to deal with the large state space in an SDP.

5.3.7 Other approaches

Other successful approaches are not classified within any of the aforementioned methods. Some of these approaches can be pointed out as game theoretical analysis using Nash equilibrium (Büyüktahtakın et al. 2011b), the Shapley value (Büyüktahtakın et al. 2013), neural networks (Potapov 2009), Lagrangian relaxation (Mbah and Gilligan 2010), and statistical approaches such as spatial sampling (Demon et al. 2011).

6 Concluding remarks and future directions

Invasive species are a major threat to our environment, society, and economy because they threaten biodiversity, increase human health problems, reduce the value of agricultural products, and endanger life, property, and safety due to increased fire risks, while costing billions of dollars each year for management efforts. The international community, including the Global Invasive Species Program (GISP), formed by the United Nations, and other international organizations, such as the National Invasive Species Council and the Center for Invasive Species Management (CISM), stress the importance of rapid control of invasive species in order to minimize their adverse impacts. Given that the funding allocated to controlling invaders is extremely limited, utilizing the available resources as effectively as possible is a highly complex problem, requiring the use of OR as a decision-support tool. OR decision tools can help managers assess alternatives, prioritize investment, and make better decisions in the battle against invaders. In this paper, we have reviewed a range of OR methods in invasive species management and discussed their ability to address some of the major challenges of biological invasions, their spatio-temporal management, mathematical complexity, and solution of the models. We have also highlighted a plethora of challenges, gaps, and future directions in the literature regarding invasive species control in this section.

Evidence from this study suggests that ISM modeling has mostly received the attention of researchers in domain areas such as ecology, biology, environmental sustainability, and resource economics. Although some logistical considerations have already been incorporated into ISM approaches, the OR community could still provide useful input in ISM, in particular, on the analysis of ISM logistics and coordination. Furthermore, the complexity of OR models in ISM and the need for exact optimal solutions for large-scale instances would require the contribution of mathematicians and OR researchers in developing advanced solution algorithms to tackle such complex models.

This review also highlights the fact that OR could play an important role in invasive species management and environmental decision-making, particularly in closing the gap between the decision-support needs of managers and the decision-support tools currently available for ISM. The successful development and use of OR methods as decision-support tools will require the OR analyst to work closely with field researchers, conservationists, as well as practitioners (see, e.g., Church et al. (2000)). Such interactions would facilitate the development of data-driven models that could offer practical guidance to inform policy decisions.

Following the insights derived from the analysis of a large variety of modeling studies in ISM, we propose many opportunities for future research investigation. More specifically, future research directions may include the following:

- **More realistic assumptions** Future research approaches should incorporate more realistic features regarding the population dynamics of invaders. For example, biological factors such as growth and dispersal are critical parameters in invader control, and therefore, future OR modeling approaches should take into account invasion dynamics when considering logistical considerations. Rapid containment of biological invasions is another critical factor, thus necessitating models that consider minimizing the control time. In addition, studying spatially, temporally, and structurally (aged or density based) changing rates in the optimization models is important to improve the realism of those models. There is also a gap between policy implementations and modeling approaches, and therefore, researchers are encouraged to collaborate with practitioners while following standards and guidelines published by environmental and governmental organizations when developing ISM mathematical models. For example, in order to ensure compliance with local, national, and regional regulations, additional constraints, such as monitoring and quarantine requirements, could be integrated in ISM optimization models. Incorporating these considerations while taking into account more realistic assumptions would lead to more robust models.

- **Combined management strategies** Relatively fewer studies have investigated the resource allocation among joint intervention efforts such as prevention, surveillance, and control in ISM. Most of the modeling approaches including multiple intervention focus on SDP and MDP. Furthermore, the bioeconomic realism considered in those models is limited. Future research could investigate biologically realistic models for the optimal resource allocation among multiple management strategies. In particular, mathematical programming tools, such as MINLP and MIP, could prove useful in prioritizing the management efforts of invasive species over multiple sites on a multi-period planning horizon.
- **Uncertainty** The major uncertainties in ISM include, but are not limited to, the following: (a) uncertainty in introductions and establishments, (b) dispersal and state transition uncertainty, (c) uncertainty in current extent of the invasion and species presence, (d) uncertainty in natural disturbances, and (e) uncertainty in available resources. In addition, due to environmental variation and human error, the effectiveness of management strategies, such as detection, successful eradication, and containment, is uncertain.
  Future OR models that embed stochastic parameters would provide more robust models for practical use in ISM (Cobuloglu and Büyüktahtakın 2017). The majority of OR approaches in ISM use SDP techniques, while the use of other stochastic optimization methods is limited. To this end, analytical methods such as stochastic programming, robust optimization as well as chance-constrained and scenario-based optimization could be used to handle mathematical optimization models with uncertain parameters, in addition to SDP, MDP, and simulation approaches.
- **Logistic considerations** Previous work on spatial ISM focuses on long-term strategic decisions such as where and when treatment should be applied over multiple years, while short-term operational decisions have not received sufficient attention from researchers. Future research should analyze operational decisions, such as allocation of the treatment crew, equipment, and other resources among different treatment sites, and the routing of the crew between selected sites for management. Deployment of multiple resources such as funding and labor among different treatment options (e.g., mechanical control, herbicide treatment, prescribed burning) over multiple sites and time periods is also another possible future direction. Scheduling of actions such as containment, surveillance, and control as well as the allocation of tasks among personnel are other possibilities for future investigation.
  Another interesting future direction in ISM is the coordination of capacity (e.g., personnel and equipment) among stakeholders. In particular, cooperation among independent but related parties to share their resources, capacities, and information could improve the efficiency of ISM. OR models have been widely used for supply chain coordination among stakeholders in production and manufacturing, disaster management, and bioterrorism response (Altay and Green III 2006; Lee et al. 2006; Thomas and Griffin 1996).

Similar models could be utilized for ISM resources management and coordination among managers at the federal, state, and local level.
Transportation and trade also increase the risk of biological invasions. For example, many wood-boring or bark-dwelling insects, such as emerald ash borer, are transported through eggs or larvae inside wood and wood packaging. Pest movement in wood packaging is further facilitated by faster transport and the use of shipping containers (Hulme 2009). Future studies could incorporate the risks of invasion related to transportation in an optimization model in which the routes, through which wood, feedstock, and propagules are transferred, are optimally selected while minimizing the distances that potential invaders are transported. Discrete and continuous network optimization models, such as shortest path and minimum cost network flow problems (Ahuja et al. 1993), can be used to formulate the transportation of goods that pose risk of invasion. Future research could also optimize the design of the transportation network and the selection of appropriate means for transportation and distribution activities while minimizing the risk of introduction and establishment of new invaders.

- **Model complexity** The computational complexity of OR models for ISM applications has remained the biggest challenge to implementing these models in computational software and obtain optimal solutions. Difficulties with the ISM models include the formulation of nonlinear biological intricacies, uncertainties in vital rates of the species, economic restrictions, as well as the computational burden caused by the complexity of modeling spatial heterogeneity and temporal dimensions in such a biological system (Kibis and Büyüktahtakın 2017). Even without considering the complicating biological factors, ISM is a spatio-temporal resource allocation problem, which is shown to be NP-complete in the strong sense (Kellerer et al. 2004). Most previous work studies the application of the invasive species control problem to a real-life problem without focusing on the solvability issues of the complex optimization problem.
Mathematical programming is a suitable tool to handle various model complexities including spatial heterogeneity and uncertainty. For example, the mixed-integer programming approach provides numerous advantages compared to specialized solution algorithms including the simplicity of implementation of advanced MIP solver software, reliability of the method, exact or guaranteed solution of the problem, and ease of modifying the model (Billionnet 2013).
Despite the availability of advanced optimization solvers such as CPLEX, the spatio-temporal characteristics significantly increase the complexity of the MILP and nonlinear MILP models, making the solution intractable. Future research should investigate decomposition algorithms to tackle large-size MILPs in order to solve the full practical-size ISC problem. Furthermore, cutting-plane methods exploiting the special structure of the problem could be developed in order to improve the effectiveness of formulations (Nemhauser and Wolsey 1988b).
- **Data and parameterization** One important gap in the literature is the integration of OR models and large-scale empirical data. Models driven by field data are particularly powerful for understanding the impact of biological characteristics as well as economic realities in guiding management decisions. Such modeling efforts require the collaboration of OR analysts with field biologists. For example, biologists could collect field data in order to quantify germination, survivorship among age stages, and seed production. The mathematical modeler then analyzes and parametrizes the collected large-scale data using statistical methods. The parametrized data is then used as an input into a mathematical model. ISM modeling studies commonly assume homogenous data, in which parameter values do not vary among different spatial locations or over different points

of time. A model with spatially and temporally heterogeneous data would provide more realistic recommendations compared to others using limited, homogenous or synthetic data.
Uncertainties in environmental parameters are infrequently handled in the literature (Beale and Lennon 2012; Rivington et al. 2006). However, data uncertainties affect output from all models and therefore need to be incorporated in model predictions in order to better inform management actions. One particular challenge is the collection of empirical data and the quantification of uncertainty and spatiotemporal variation in vital rates such as intrinsic growth rate, offspring production, carrying capacity, dispersal uncertainty, and the likelihood of detection. The quantification of uncertainty requires the estimation of probability distributions and the use of statistical methods, while uncertainty analysis could be performed using Monte Carlo simulations, sampling methods, and regression (Hammonds et al. 1994). While various sources of uncertainty have been previously identified and incorporated into ISM mathematical models to some extent, future work could measure and study all known sources of uncertainty comprehensively in one mathematical model (Beale and Lennon 2012). Furthermore, the effect of uncertainties on modelled predictions could be investigated in a future study (Elith 2013).

- **Validation of OR models in ISM** To our knowledge, OR models in controlling invasive species have not been validated yet. Therefore, future research should assess the applicability of the existing ISM modeling frameworks to real settings. Validation of such models could be achieved by face validation through evaluating expert opinion on the inputs and outputs of the model, and validation based on field experiments in order to confirm model predictions by field data (Gass 1983). Such evaluations could validate the adaptability of existing models as well as the suitability of methodologies applied. Validation efforts could also provide useful insights and input for the refinement of existing frameworks and development of more robust approaches.
- **Economic assessment of the benefits of ISM** One of the most important components in the determination of optimal ISM strategies is an economic assessment of the benefits (i.e., avoided damage) of management activities (see objective function, Eq. 3). Benefits depend on the biological characteristics of the species (e.g., likelihood of establishment, lags, and rates of spread) as well as the long-term damage if the invasion were to occur (Epanchin-Niell and Liebhold 2015). The long-term damage reduction associated with ISM strategies is difficult to quantify, especially at the regional or national scale (Aukema et al. 2011). Methodological challenges include understanding and modeling the complex dynamics of invasion and damage processes (Kovacs et al. 2010; Soliman et al. 2012) and estimating the value of damages to ecosystem services, such as water and air quality, nutrient cycling, climate regulation, and recreation, that are not traded in markets. Aukema et al. (2011) begin to address these issues with a study of the economic impacts of 455 non-native forest insect species known to be established in the continental United States.
- **Coordination among multiple stakeholders** Biological invasions, by their nature, cross jurisdictional boundaries as they spread, and control of the invasion often depends on the choices of many decision-makers across the landscape. Each landowner typically decides how intensively to manage an invader based on local damage and management costs, without considering the benefits of control for neighbors and the general public. As a result, a landowner acting independently is likely to underinvest in control from a societal perspective. Economists have generally found that cooperative or centralized control of a biological invasion across jurisdictions is superior to independent management (Büyüktahtakın et al. 2013; Epanchin-Niell and Wilen 2015; Kovacs et al. 2014; Aadland et al. 2015; Büyüktahtakın et al. 2011b). A mechanism of transfer payments in

which one jurisdiction pays another to increase their level of control (Bhat and Huffaker 2007) is one method of cooperation. In other situations, jurisdictions may simply agree to coordinate their efforts in a beneficial way to minimize spillover effects (Epanchin-Niell and Wilen 2015). Cooperative game theory, such as the solution concept of Shapley, is used to determine compensation efforts and optimal cooperation among multiple stakeholders (Büyüktahtakın et al. 2013). The design of market-based and regulatory policies to enhance cooperation across jurisdictions is a key area for further research.

- **Need for holistic approach** Invasive species programs cover a wide range of management options, all of which compete for public resources. The tradeoffs between program costs and economic losses are often poorly understood, and this lack of information makes it difficult to design cost-effective programs. Efficient allocation of public resources requires a framework that links prevention, surveillance, and control programs and their effects on population dynamics and damages across a range of potential or current invaders. Further, what is needed is an integrated framework that links human processes governing long-distance movement (e.g., trade); biological processes governing establishment, local spread, and consequences for ecosystem services; and human processes governing the value of those ecosystem services. Such a framework could then be used to evaluate the benefits and costs of alternative allocation strategies and suggest optimal strategies.

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Appendix

Here we provide Table A that summarizes a classification of studies with respect to various types of complexities, size of application, and solution methodology, and Table B that classifies studies with respect to the objective of the study, application considered, and specific conclusions.

Table A Classification of studies with respect to various types of complexities, size of application, and solution methodology

Category	Study Ref (alphabetically ordered in each category)	Model Type			Biological Complexity						Model Complexity				Mathematical Complexity				Size of Application		Modeling and Solution Method
		Prevention	Surveillance	Control	Growth	Seedbank	Proportion or State	Dispersal	Carrying Capacity	Stage-structured	Budget const	Spatially explicit	Temporal	Uncertainty	Linear	Non-linear	Integer	Non-linear Integer	Spatial	Temporal	
Control	Aadland et al. 2015			x	x			x				x	x			x			6 × 6 grid	T=200	First-order Approximation, Simulation, Rolling Horizon
	Albers et al. 2010			x	x			x	x			x	x			x			Two regions	Undefined	Optimal Control, Differential Equations, Numerical Analysis
	Baker and Bode 2013			x	x			x			x		x			x			One region	Undefined	Optimal Control, Numerical Analysis
	Baxter et al. 2007			x	x	x	x						x	x	x				One region	T=20	SDP
	Bhat et al. 1993			x	x			x	x			x	x			x			6x10 grid	T=30	Differential Equations, Numerical Analysis, Rolling Horizon, Simulation
	Blackwood et al. 2010			x	x			x				x	x		x				30x30 patches	T=15	Linear-Quadratic Control, Optimality Conditions
	Bogich and Shea 2008			x	x								x	x	x				One region	T=10	SDP
	Büyüktahtakın et al. 2011a			x	x			x	x		x	x	x					x	40x50 grid	T=30	Rolling horizon, MINLP
	Büyüktahtakın et al. 2013			x	x			x	x		x	x	x					x	40x50 grid	T=30	Game theory, Shapley Value
	Büyüktahtakın et al. 2014a			x	x			x	x		x	x	x					x	40x50 grid	T=30	Multi-Objective Optimization
	Büyüktahtakın et al. 2015			x	x	x		x	x	x	x	x	x	x				x	10x10 grid	T=45	MINLP, Rolling horizon
	Eiswerth and Johnson 2002			x	x				x				x			x			One region	Undefined	Optimality Conditions
	Finnoff et al. 2010			x			x	x					x			x			One lake	T=5	Differential Equations, Optimality Conditions
	Firn et al. 2008			x		x	x	x					x	x	x				One region	T=10	SDP
	Grimsrud et al. 2008			x	x			x	x				x			x			One region	T=5	Non-linear Differential Equations, Simulation
	Haight et al. 2011			x	x					x		x	x	x	x				1240 grid cells	T=10	Stochastic Simulation
	Hof 1998			x	x			x			x	x	x		x				30x30 grid	T=5	Linear Programming
	Hof and Bevers 2000			x	x			x			x	x	x		x				900 cells	Undefined	Linear Programming
	Hyder et al. 2008			x	x		x	x	x				x	x		x			2000 × 2000 grid	T=81	SDP
	Kaiser and Burnett 2010			x	x			x	x		x	x	x					x	38,000 grid cells	T=30	MINLP, Genetic algorithm, Myopic Policy Solution
	Kibis and Büyüktahtakın 2017			x	x	x		x	x	x	x	x	x	x				x	10x10 grid	T=7	MINLP, Linearization, MIP
	Sebert-Cuvillier et al. 2008			x			x	x				x	x	x	x				41x29 cells	T=152	Simulation
	Taylor and Hastings 2004			x	x					x	x		x			x			One region	T=60	Genetic Algorithm
	Yakob et al. 2008			x	x			x	x	x		x	x	x		x			500 nodes	T=50	Stochastic Network Simulations
	Yokomizo et al. 2009			x	x				x		x		x			x			One region	T=5	SDP
Surveillance or Surveillance & Control	Baxter and Possingham 2011		x				x	x			x		x	x	x				1000 grid cells	T=20	SDP, Simulation
	Bogich et al. 2008		x	x	x								x	x	x				One region	T=20	Numerical Analysis
	Cacho et al. 2007		x	x	x	x	x			x			x	x	x				One region	T=60	Numerical Analysis
	Cacho et al. 2010		x	x			x	x				x	x	x	x				128 x128 grid	T=15	Simulation
	Demon et al. 2011		x				X	x					x			x			One region	T = 100 d	Spatial Sampling, Simulated Annealing
	Epanchin-Niell et al. 2012		x	x	x					x	x		x	x		x			One region	T=4	Constrained Optimization, Interior Point Algorithm
	Epanchin-Niell et al. 2014		x		x			x		x	x	x	x	x		x			4 port regions	T=100	Numerical Analysis
	Haight and Polasky 2010		x	x			x						x	x	x				One region	T=20	POMDP
	Hauser and McCarthy 2009		x	x			x				x		x	x		x			One region	T=1	KKT Optimality Conditions
	Homans and Horie 2011		x	x	x							x				x			One region, T=1	T=1	Stochastic Differential Equations, Numerical Analysis
	Horie et al. 2013		x	x	x		x				x	x		x			x		90 Hexagonal grid	T=10 y	Scenario-based MILP
	Mehta et al. 2007		x	x	x								x	x		x			One region	T=100 h	Optimality Conditions
	Moore and McCarty 2016		x								x	x	x	x			x		19 sites	T=55.2 h	2-Factor Approximation
	Pichancourt et al. 2012		x	x	x	x		x		x			x		x				One region	T=5	SDP, MDP
	Regan et al. 2006		x	x			x						x	x	x				One region	T=44	SDP

Table A continued

Category	Study Ref (alphabetically ordered in each category)	Model Type			Biological Complexity						Model Complexity				Mathematical Complexity				Size of Application		Modeling and Solution Method
		Prevention	Surveillance	Control	Growth	Seedbank	Proportion or State	Dispersal	Carrying Capacity	Stage-structured	Budget const	Spatially explicit	Temporal	Uncertainty	Linear	Non-linear	Integer	Non-linear Integer	Spatial	Temporal	
Prevention or Prevention & Control	Burnett et al. 2008	x		x	x				x				x			x			One region	T=10 y	Hamiltonian (Control Theory), Optimality Conditions
	Chen et al. 2017	x			x						x			x		x			One region	T=1	Kuhn-Tucker Conditions
	Epanchin-Niell and Wilen 2012	x		x			x	x				x	x				x		15x15 grid	T=17	MILP
	Finnoff et al. 2007	x		x	x				x				x	x		x			One region	Undefined	SDP
	Hof et al. 1997	x		x	x			x	x			x	x				x		5x5 grid	T=4	MILP
	Kovacs et al. 2014	x		x	x			x			x	x	x			x			17 cities	T=5	Nonlinear Programming, Optimality Conditions
	Leung et al. 2002	x		x	x		x	x		x	x		x	x		x			One region	T=10	SDP
	Moore et al. 2011	x		x	x		x						x	x		x			One region	T=20	Numerical Analysis
	Olson and Roy 2005	x		x			x						x	x		x			No case study	Undefined	Differential Equations, Optimality Conditions
	Potapov et al. 2009	x		x	x			x				x	x	x		x			14 lakes	Undefined	Neurodynamic Programming, Neural Networks
	Sharov and Liebhold 1998	x		x	x			x	x	x		x	x			x			One region	Undefined	Numerical Analysis
	Surkov et al. 2009	x									x		x	x		x			One region	T=10 y	Partial-equilibrium
	Springborn 2014	x					x				x	x	x	x		x			2 inspection sources	T=2	Approximate dynamic programming
Prevention & Surveillance & Control	Carrasco et al. 2010	x	x	x	x			x			x		x	x		x			One region	T=20	Simulation, Genetic Algorithms
	Hyytiäinen et al. 2013	x	x	x		x	x		x	x			x	x	x				One region	T=100	SDP
	Polasky 2010	x	x	x			x						x	x		x			One region	T=100	SDP
	Rout et al. 2011	x	x	x			x						x	x		x			One region	T=10	SDP
	Rout et al. 2014	x	x	x			x						x	x		x			One region	T=10	POMDP
	Mbah and Gilligan 2010	x	x	x	x		x	x	x		x		x			x			One region	Undefined	Lagrangian Relaxation, Hamiltonian Method
	Moore et al. 2010	x	x				x						x	x	x				One region	T=20	SDP, MDP

Table B Classification of studies with respect to objective, application, and specific conclusions

Category	Study Reference	Objective/Focus	Application					Specific Conclusions
			PL	PE	AQ	AN	Species Name, Location	
Control	Aadland et al. 2015	Maximize society's welfare from the landscape forest, while evaluating tree harvesting rules that incorporate spatial risks from insect outbreaks		x			Mountain pine beetle, unspecified	Harvesting dampens outbreaks but causes them to spread faster over the landscape. The nonlinear problem cannot be solved for even a 2x2 spatial grid.
	Albers et al. 2010	Maximize economic welfare from foreign and domestic trade across the two regions, while exploring the costs and benefits of centralized and decentralized policies					Hypothetical	Optimal uniform strategies can differ substantially among the spatial and uniform scenarios.
	Baker and Bode 2013	Minimize the total costs of bait applied and remaining predators at the conservation asset while identifying the best landscape distribution of bait				x	Feral foxes/doves, unspecified	It is optimal to place the highest bait concentration near the conservation asset. The optimal baiting strategy in two dimensions results in a larger improvement over uniform baiting than in one dimension.
	Baxter et al. 2007	Minimize the total expected costs of weed removal, while finding the optimal proportion of plants to remove	x				Annual weed, unspecified	The optimal removal effort increases non-linearly with the density of plants. The solution is most sensitive to population growth rate, the escape probability function, and the relative costs of escape and management.
	Bhat et al. 1993	Minimize the sum of economic and ecological loss due to beaver-inflicted timber damage and beaver-trapping cost, while accounting for migration of the species				x	Beaver, State of New York	Increased trapping in the initial years of the program results in a smaller number of beavers trapped over the entire time horizon.
	Blackwood et al. 2010	Minimize the present discounted cost of invasive species, while determining optimal eradication using transition matrices to formulate dispersal among patches	x				Spartina alterniflora, San Francisco, CA	Early removal is suggested to prevent an excessive increase in ecological damage as a result of an expanding invasion. Reducing connectivity can be a cost-effective part of invasive species control.
	Bogich and Shea 2008	Minimize the number and density of infested patches ahead of the invasion front, while determining general rules of thumb for dealing with pest infestation		x			Gypsy moth, North America	The optimal strategy for gypsy moth management include two points: (1) eradicate medium patches and (2) reduce large patches to medium patches.
	Büyüktahtakın et al. 2011	Minimize the damage weighted by the value of impacted resources, while explicitly formulating invader nonlinear growth, and where and when to apply treatment	x				Buffelgrass, Arizona	There is a need for a policy change, since the current resources cannot stop the emerging ecological disaster in the near future. Consecutive treatment obtained by considering both current and future damages is the best control strategy.
	Büyüktahtakın et al. 2014a	Minimizes the damage to three different valued and threatened resources including saguaros (a native cactus species), buildings, and vegetation	x				Buffelgrass, Arizona	Cooperation of the different interest groups is absolutely essential in establishing reasonable treatment strategies; otherwise, the total damage becomes very large.
	Büyüktahtakın et al. 2013	Determine what payoff will be awarded to each decision maker in a cooperation by using cooperative game theoretical methodology	x				Buffelgrass, Arizona	A homeowner strategy of protecting against wildfire affords less protection to the other resources. Under the optimal solution, groups caring about riparian vegetation would compensate homeowners and groups caring about saguaros.
	Büyüktahtakın et al. 2015	Minimize the total economic damages caused by an invasive species over space and time, while explicitly formulating seed-bank-based growth and seed dispersal	x				Sericea Lespedeza, Great Plains	Age and spatial structure leads to multilogistic population growth. Given budget constraints, utilizing control measures every 2–3 years is found to be more effective than yearly control because of the time to reproductive maturity.
	Eiswerth and Johnson 2002	Minimize the total net economic damages that occur due to an invasive species stock, while determining the optimal removal strategy	x				Multiple species (e.g., Tamarisk, cheatgrass), Great Basin	Optimal management effort is sensitive to the stock's intrinsic rate of growth, carrying capacity, and form of the invader's growth function, which are species- and site-specific as well as uncertain.
	Finnoff et al. 2010	Maximize the discounted stream of social welfare, while determining the optimal intensity of control			x		Zebra mussel, across lakes in Wisconsin	Control to a highly invaded steady state is optimal for those invasions with low relative damages, high chances of random introduction, and high levels of uncertainty in species location.
	Firn et al. 2008	Maximize the number of sites occupied by native vegetation, while exploring how asymmetries in disturbance between native and invaded sites affect optimal removal	x				Mimosa pigra, perennial shrub, Australia	When disturbance to native vegetation is high, optimal control focuses on reducing the invader seed bank. Otherwise, control efforts shift from attempting to reduce the size of the seed bank to focusing on killing the current population.

Table B continued

Category	Study Reference	Objective/Focus	Application: PL	PE	AQ	AN	Species Name, Location	Specific Conclusions
Control	Grimsrud et al. 2008	Maximize net benefits, while an agent "i" chooses the optimal effort level, given the observed actions of agent "j" and the initial rangeland conditions	x				Yellow Starthistle, New Mexico	Coordinated efforts among agents are required to minimize impact. The level of infestation impacts the amount of incentive necessary to get the agent to control weeds, and the incentives impact the level of effort of the rancher.
Control	Haight et al. 2011	Estimate the baseline economic damage from an invasive pathogen, a significant fungal disease of oaks that causes oak wilt		x			Ceratocystis fagacearum causing oak wilt, Minnesota	There are significant economic benefits, in terms of damage reduction, from controlling the establishment of new pockets or slowing the radial growth of existing pockets.
Control	Hof 1998	Minimize the total exotic pest population by incorporating a management effort that is directly related to pest mortality		x			Hypothetical	The management strategy is to focus efforts outside the limited host area and to take advantage of its inhospitable conditions or the pest.
Control	Hof and Bevers 2000	Minimize an undesirable exotic pest population, while determining the spatial amount of management effort		x			Hypothetical	A range of spatial problems can be directly modeled with linear, continuous-variable formulations. An optimization methodology is necessary to make progress, either in learning about the ecological system or in managing it.
Control	Hyder et al. 2008	Maximize the welfare due to the reduced amount of infestation, while determining the optimal strategy of whether to apply biological or herbicide control	x				Leafy spurge (Euphorbia esula), North Dakota	The optimal strategy depends on the area, density, and planning horizon; therefore, dynamic control is important in management programs. Biological control is consistently the optimal strategy.
Control	Kaiser and Burnett 2010	Minimize the present value of costs and damages for optimal removal of the snake invasion				x	Brown treesnake (Boiga irregularis), Hawaii	A more aggressive, yet carefully targeted, search is preferred to a point, but aggressive policies that are random in nature are more costly than either optimal or myopic strategies.
Control	Kibis and Büyüktahtakın 2017	Minimize the economic damage associated with invasive species under a limited treatment budget while taking into account its spatial dynamics	x				Sericea Lespedeza, Great Plains	While MINLP could not solve the spatially explicit model for a 3-by-3 grid landscape with two time periods, the proposed MIP solves the problem with a 10-by-10 grid landscape with a seven-year period for the first time.
Control	Sebert-Cuvillier et al. 2008	Using simulation, examine invasion dynamics, such as spatial heterogeneity, seed dispersers, site of first introduction, and forest management	x				Alien tree species, Prunus serotina Ehrh., France	Spatial heterogeneity increases the invasion speed but decreases the final invasion extent. The site of initial introduction and natural disturbances (such as severe storms) influences the invasion process.
Control	Taylor and Hastings 2004	Minimize the total effort and risk of colonization of other sites within the bay, while finding optimal control strategies	x				Spartina alterniflora, Washington State, USA	The optimal strategy depends on the annual control budget. At low and medium budgets, low-density plants are given priority for eradication, but if more money is available, then the optimal strategy is to prioritize high-density areas.
Control	Yakob et al. 2008	Using stochastic network simulation, determine the effect of the insect's migration rate and dispersal distance on the projection of its control		x			Hypothetical insect	When insects are non-randomly distributed across a landscape, control can be significantly hindered. However, when insect populations are clustered as a result of limited dispersal, genetic control efficiency can be improved.
Control	Yokomizo et al. 2009	Minimize the total cost of impact and management, while determining the effects of population dynamics and density-impact curves on optimal management effort			x		Hypothetical	For species that are only problematic at high density, ignoring the density-impact curve can lead to excessive investment in management with little reduction in impact.
Surveillance or Surveillance & Control	Baxter and Possingham 2011	Examine under which cases managers should adopt one of two search strategies (cursory or focussed) for allocating prediction funds		x			Fire ant, Queensland, Australia	Widespread searching is only optimal if the pest is already widespread or knowledge is poor; otherwise focused searching is preferable. For longer management timeframes, increased initial knowledge improves eradication
Surveillance or Surveillance & Control	Bogich et al. 2008	Minimize both the search-and-control costs of a gypsy moth infestation by determining the optimal trap density for detecting isolated infestations		x			Gypsy moth (Lymantria dispar), North America	Optimal trap densities are lowest for infestations with very low growth rates or very high growth rates (because they are easier to detect), and highest for infestations with moderate growth rates (because they are not easy to detect).
Surveillance or Surveillance & Control	Cacho et al. 2007	Minimize the sum of search, control, and program administration costs, and analyze the significance of monitoring and detectability on successful eradication	x				Mesquite, Galapagos Islands	The success of an eradication program depends critically on the detectability of the target plant, the effectiveness of the control method, the labor requirements for search and control, and the germination rate of the plant.
Surveillance or Surveillance & Control	Cacho et al. 2010	Minimize the cost of search, while simulating the dispersal of invasive species, and considering both active and passive surveillance efforts					Hypothetical	Increased passive surveillance reduces control costs significantly and increases the probability of eradication as active surveillance is enhanced. Even small increases in detection or reporting rates substantially reduce eradication costs.

Table B continued

Category	Study Reference	Objective/Focus	Application: PL	PE	AQ	AN	Species Name, Location	Specific Conclusions
Surveillance or Surveillance & Control	Demon et al. 2011	Maximize the probability of detecting an invasive pathogen by determining optimal location for a sample in a spatially explicit epidemic model		x			P. ramorum, Devon, UK	If the location of the source of infection is known, then distance-based sampling is a simple and efficient method with a probability to detect disease that is close to the maximum, as provided by optimized spatial sampling.
	Epanchin-Niell et al. 2012	Minimize the total cost of surveillance and control, while enabling multiple surveillance efforts over time		x			Gypsy moth (Lymantria dispar), California	A higher surveillance effort is required for invasive species that have higher growth and dispersal rates, and are more costly to eradicate.
	Epanchin-Niell et al. 2014	Minimize the total expected costs of invasion, including the costs of surveillance, invasion control, and damages, by determining the numbers and distributions of traps.		x			Invasive wood borer and bark beetles, New Zealand	Even low levels of surveillance are useful. The greatest payoffs from surveillance occur for large amounts of imports and in areas where damages will be high due to the proximity to high-value, at-risk resources.
	Haight and Polasky 2010	Minimize expected costs under infestation uncertainty by updating beliefs about the state of the infestation and the value of improved information through monitoring.					Hypothetical	The optimal policy involves employing no action when the probability of no infestation is sufficiently large, monitoring alone with medium probability, and treatment alone when the probability of moderate or high infestation is large.
	Hauser and McCarthy 2009	Minimize expected management costs by determining the surveillance strategy, and considering economic value and the ease of detection and control	x				Orange hawkweed (Hieracium aurantiacum), Australian	Environments where the detection is easy are prioritized for surveillance, while only a moderate investment is necessary to ensure a high probability of detection.
	Homans and Horie 2011	Minimize the total cost associated with detection, treatment, and economic damage caused by the invasive pest while choosing the optimal detection strategy		x			Gypsy moth, Undefined	A high dispersal range adversely impacts the detection of the dispersed population and results in a more aggressive approach to finding and suppressing the pest.
	Horie et al. 2013	Minimize the number of newly infected trees in the entire management area where there is uncertainty about the extent of the infestation in each site	x				Oak wilt, Minnesota, USA	More accurate estimates of the proportion of infected trees through increased sampling reduces the cost of surveillance and removal. Choosing sites with a high expected numbers of infected trees performs well relative to the optimum.
	Mehta et al. 2007	Minimize the total costs of management and the damages caused by an invasive species by capturing the stochastic and dynamic trade-offs between detection and control					Hypothetical	The optimal detection strategy depends primarily on the "detectability," or ease of detection, growth parameters, and initial population size.
	Moore and McCarty 2016	Maximize the expected number of detections of invasions by finding the optimal allocation of survey effort over space and time				x	Cascade treefrog (Litoria pearsoniana), Unspecified	Considering variable detection rates and travel costs do not impact the form of the resulting objective function, and the probability of failed detection at each site is a negative exponential function of effort.
	Pichancourt et al. 2012	Minimize the cost of controlling the core infestation locally and the future cost of control arising from infestation via seed dispersal	x				Mesquite, Australia	The most cost-effective control strategy may include targeting smaller-size classes than adults. The knowledge of the density of larger trees is likely to be more important than smaller more cryptic-size classes for optimal control.
	Regan et al. 2006	Minimize expected costs, while answering the question of how many years we should survey and not detect the species before stopping, using a stochastic process model	x				Helenium amarum, Queensland, Australia	Given that the detection of a species is not perfect, the optimal stopping time is a trade-off between the cost of continued surveillance and the cost of escape and damage if eradication is declared too soon.
Prevention or Prevention & Control	Burnett et al. 2008	Minimize expected damages to the state's ecological assets and economy, while analyzing optimal resource allocation for prevention and control simultaneously		x			Brown treesnake, Hawaii	It is more advantageous to spend money for detecting the small population of snakes than concentrating almost all funding on prevention.
	Chen et al. 2017	Minimize the costs of pest introductions from trade by posing inspection as an acceptance sampling problem under a limited budget and heterogenous lots	x				Ninety-one plant species, US	Allocating any additional capacity to the largest lots with the highest plant infestation rates allows to minimize the costs of slippage and maintain baseline sampling.
	Epanchin-Niell and Wilen 2012	Minimize the expected net present value of total costs and damages, while determining binary treatment and prevention efforts					Hypothetical	System-wide invasion externalities usually increase with the potential range size of an invader and with marginal damage from invasion, and decrease with marginal control costs and the size of the invasion when it is discovered.
	Finnoff et al. 2007	Maximize social welfare, while analyzing the tradeoffs between prevention and control, incorporting risks			x		Zebra mussels (Dreissena polymorpha), North America	Greater aversion to risk leads to less prevention and greater control, which increases the probability of invasions and realized abundance of invaders, thus lowering the overall social welfare in comparison to less risk aversion.

Table B continued

Category	Study Reference	Objective/Focus	Application					Specific Conclusions
			PL	PE	AQ	AN	Species Name, Location	
Prevention or Prevention & Control	Hof et al. 1997	Use spatial optimization for managing an aggressively dispersing pest, while limiting the host capacity accumulated over time		x			Hypothetical	Both formulations with and without host capacity are difficult to solve.
	Kovacs et al. 2014	Maximize benefits from healthy ash trees over a planning horizon, while optimally treating or removing ash tree phloem to reduce the emerald ash borer (EAB) infestation		x			Emerald ash borer, Minnesota	Net benefits can be maximized by making a cooperative decision. Treatment of trees is superior to their removal because treatment reduces the amount of phloem at a lower cost.
	Leung et al. 2002	Maximize welfare, while analyzing optimal allocation of resources for prevention versus control, and acceptable invasion risks			x		Zebra mussels (Dreissena polymorpha), lakes in Michigan	A much higher value should be placed on prevention than is currently spent. Greater prevention will protect the environment while also protecting industry.
	Moore et al. 2011	Minimize total costs of management and production losses, while identifying whether eradication or containment is the most cost-effective management	x				Acacia paradoxa DC, South Africa	Incorporating uncertainty in the analysis avoids overly optimistic beliefs about the effectiveness of management. Eradication is likely to be cost-effective, particularly if resources are allocated to improve management.
	Olson and Roy 2005	Minimize the expected social costs from prevention, control, and invasion damages, while studying the appropriate balance between the prevention and control					Hypothetical	An increase in the variability of introductions increases the marginal benefits of prevention and control. The marginal damage function plays an important role in determining the impact of invasive species introductions.
	Potapov et al. 2009	Minimize the current and weighted future costs, while obtaining an approximate control policy for an aquatic invader that is transported along with recreation boats			x		Hypothetical	The intensity of control depends on the trade-off between losses caused by invasion and cost of the control procedures. There is a critical value of losses, below which it is always optimal not to control at all.
	Sharov and Liebhold 1998	Predict the effect of barrier zones on the rate of gypsy moth spread while assuming the establishment of isolated colonies beyond the expanding population front		x			Gypsy moth (Lymantria dispar L.), North America	The reduction of colonization rate has a very limited effect on the rate of population spread, and thus quarantine regulations may not always be an efficient mean to reduce the rate of population spread.
	Surkov et al. 2009	Minimize total societal costs of pest losses and inspection, while deciidng on the optimal level of inspection	x				Chrysanthemums, Netherlands	A budget increase that enables forty-two percent more inspection can reduce total societal costs by eighty-one per cent
	Springborn 2014	Maximize the total expected payoffs over an infinite horizon subject to an inspection budget, while detrmining the number of shipments to inspect					Hypothetical, US	The decision maker may accept the most likely outcome is a small net loss because the exploratory approach buffers against potential large losses
Prevention & Surveillance & Control	Carrasco et al. 2010	Minimize the net present value of total costs due to invasions and their management while considering the exclusion, detection, and control of multiple invaders		x			Colorado beetle, UK	Agencies should allocate less exclusion and more control resources to NIS characterized by Allee effects and a low rate of satellite colonies generation, and that present low propagule pressure.
	Hyytiäinen et al. 2013	Minimize costs from damage and measures, while investigating time and extent of efforts to control and eradicate a newly established population			x		Asian clam (Corbicula fluminea), Baltic Sea	The results highlight the need for the energy sector to identify and internalize the external costs of potential invasions when making any large-scale investment plans.
	Polasky 2010	Maximize the present value of benefits minus the cost of prevention, detection, and control, while analyzing the trade-offs among prevention, surveillance, and control					Hypothetical	Prevention and detection are substitutes. Reducing the cost or increasing the effectiveness of one of these policies will increase the optimal effort devoted to it. Once an invader is present, control and detection are complementary.
	Rout et al. 2011	Minimize the total cost of management and impact, while identifying the trade-offs inherent in prevention, surveillance, eradication				x	Black rat (Rattus rattus), Barrow Island, Western Australia	The optimal management action depends on the effectiveness of each action and different stages of invasion. If the pest is currently absent, then it is more cost-effective to prevent impacts through prevention or surveillance.
	Rout et al. 2014	Minimize the total cost of management and impact, while determining how to allocate resources among prevention, surveillance, and eradication under uncertainy				x	Black rat (Rattus rattus), Barrow Island, Western Australia	While it is never optimal to invest solely in surveillance to reduce uncertainty, it is often optimal to combine surveillance with quarantine or control.
	Mbah and Gilligan 2010	Maximize the density of healthy individuals of two host species by controlling a common pathogen subject to a budget constraint using an epidemic model		x			A pathogen oomycete (Phytophthora ramorun) host: bay laurel and coast live oak	An intermediate level of detection is optimal. Low and high levels of detection do not bring the epidemic under control. Small changes in budget allocation may lead to drastic inefficiencies in control strategies.
	Moore et al. 2010	Minimize cost, while determining how much effort should be invested in quarantine to reduce the risk of a pest arriving vs. surveillance		x		x	Black rat (Rattus rattus), Barrow Island, Western Australia	Quarantine is optimal if large costs are associated with pest impact and as the ability to eradicate a pest declines. Surveillance is optimal if it is considerably more cost effective than quarantine, providing significant savings.

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